Frequency and Vehicle Capacity Determination using a Dynamic Transit Assignment Model

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Abstract This paper presents a model for frequency and vehicle capacity determination that uses a transit assignment model which can capture the dynamic effects on overall system performance. This allows accounting for variations in service reliability and crowding that have not been accounted for in the tactical planning insofar. Model formulation allows for minimizing user costs, operational costs or the combination of which by using simulated annealing as the search method in combination with an agent-based transit assignment simulation model. Practical benefits of the model are demonstrated by an application to a bus network in the Amsterdam metropolitan area for different demand periods, flexibility in decision variable settings and optimization objectives.

Keywords: Public transport · Frequency setting · Vehicle capacity · Transit assignment · Tactical planning

1 Introduction

The determination of frequencies and vehicle capacities is a crucial tactical decision when planning public transport services. In practice, authorities or operators typically use predefined service standards such as maximum vehicle occupancy rates as the basis for setting frequencies and vehicle capacities. Early studies proposed rule-based approaches for determining the service frequency on a line given passenger arrival rates, potential fleet size constraints and a desired maximum vehicle load factor (e.g. Salzbom 1972 and Ceder 1984). Recent approaches to this
problem use advanced optimization methods in combination with assignment models that can forecast passengers’ behavior in response to a potential supply setting. Public transport supply optimization is then solved using bi-level optimization models: an upper level supply optimization model and a lower level assignment model computing equilibrium passenger flows resulting from a certain supply given by the upper level model (e.g. Yu et al. 2010, Yoo et al. 2010, Martínez et al. 2014 Ibarra-Rojas et al. 2015). Few studies propose approaches for determining both frequencies and vehicle capacities simultaneously (Dell’Olio et al. 2012, Ruisanchez et al. 2012, Canca et al. 2016).

All methods developed so far use static assignment approaches which assume average and perfectly reliable supply conditions. Travelers are thus assumed to make decisions based on average supply conditions and performance indicators can directly be computed from the given supply and passenger flows without taking into account the dynamic interaction between demand and supply. However, especially in highly-utilized networks the dynamic and stochastic interaction between demand and supply may lead to significant reliability and crowding issues that are not accounted for in static assignment models.

The objective of this paper is therefore to develop a model for frequency and vehicle capacity determination that is able to capture the dynamic and stochastic behavior of demand and supply components in public transport networks and thus takes into account service reliability issues.

2 Method

The iterative model framework consists of three modules as illustrated in Figure 1:

(i) a dynamic public transport operations and assignment tool, BusMezzo, that considers the interaction between demand and supply and its potential impacts on service reliability (Toledo et al. 2010, Cats et al. 2016). The mutual interactions of vehicles and passengers in BusMezzo are explicitly modelled using an agent-based simulation approach with within-day and day-to-day dynamics. The latter is performed through passengers’ learning process and adaption. This iterative network loading procedure yield network-wide steady-state conditions which can be seen as an equivalent to the congested user equilibrium in conventional static assignment models. The model captured the three congestion effects in public transport networks, those are (Cats et al., 2016): (1) Deteriorating comfort on-board a crowded vehicle, (2) denied boarding in case of insufficient vehicle capacity, (3) service headway fluctuations resulting from riding and dwell time variations.

(ii) evaluating the performance of alternative solutions by transforming the outputs of the assignment model into a transport user and operator cost function. The former
is based on value of time coefficients for each passenger travel time component and the latter consists of fixed and variable costs, and;

(iii) a search algorithm that selects potential solutions using the meta-heuristic of simulated annealing, a probabilistic metaheuristic to find the global optimum in large search spaces. A neighbour of a specific solution is generated by altering either the headway or the vehicle capacity of a selected line while keeping all other variables unchanged and satisfying the feasibility constraints.

In each iteration of the optimization algorithm, a potential supply setting in terms of line frequencies and vehicle capacities is provided as an input to BusMezzo for simulation. In the process of generating new solutions, fleet availability and operational budget limitations as well as upper and lower frequency bounds can be enforced. The algorithm proceeds by selecting a random neighbour from the set and computing the objective function value using the output of the dynamic assignment model.

![Basic framework of the headway and vehicle size determination model](image)

**Fig. 1** Basic framework of the headway and vehicle size determination model

### 3 Application

#### 3.1 Case study description

The proposed model is applied to the case study network located in the ‘Zaanstreek’ area north of Amsterdam (Figure 2). The bus network consists of six lines and two different demand settings estimated based on smartcard data for the morning and the evening peak periods are analysed. The model is applied to multiple scenarios
involving different combinations regarding the formulation of the objective function, constraints and passenger demand.

Selected results are briefly presented below and the complete scenario analysis will be detailed in the full paper. On average 10 days were required to reach steady state assignment conditions per solution and 10 replications were needed in order to ensure a maximum allowable error of 1% with respect to the objective function value. The average number of search heuristic iterations required ranged between 80 and 500 whereas the solution space is approximately 1 million times larger. This yielded total running time per scenario of 40 to 240 minutes which is considered reasonable for service planning activities.

Fig. 2 Geographical (left) and schematic representation (right) of the case study bus network

3.2 Results

Figure 3 displays objective function values in terms of user and operational costs for all scenarios investigated. The results indicate that there is a clear difference between the two demand periods examined regarding both the obtained decision variables and objective function values. A minimization of total costs (TC) can yield reduction of the total system costs by up to 1.6% compared to the current situation in the morning peak period (AM), whereas in the evening peak (PM), no significant reduction can be achieved. Moreover, in the former period, all solutions tend to reduce passenger-related costs at the expense of increased operational costs, while in the latter case two of the obtained solutions are close to the current total cost composition and one solution suggests decreasing operational costs with higher user costs. In all cases, the major factor which causes changes in user costs is a reduction in waiting time. A mere minimization of total travel costs (UC) subject to a current operational budget constraint yields significant passenger benefits in both peak
periods as supply is increased up to the budget limit. The separate determination of line frequencies per direction of a route variant (ASYM) yields lower user costs compared to a conventional symmetric setting of frequencies (SYM) irrespective of the objective considered. A simultaneous determination of both vehicle capacities and line frequencies (VEHCAP) can even yield larger user benefits.

![Graph](image)

**Fig. 3** Overview of the performance of all solutions found for the different scenarios in terms of associated passenger-related and operational costs

Table 1 summarizes the obtained solutions in terms of determined headways and vehicle capacity per line obtained by the model for eight different scenarios including the base cases. Moreover, the total amount of vehicle kilometres is included as well which quantifies the total service provision. In the scenarios where total user travel costs (UC) are minimized, vehicle-km is approximately the same for all scenarios since the budget constraint is almost binding. The model finds thus solutions which all (nearly) fully exploit the budget when maximizing the benefits of passengers by allocating the amount of available resources.
Table 1 Overview of the performance of all solutions found for the different scenarios in terms of associated passenger-related and operational costs

Figure 4 shows the supply distribution in the morning peak for results of the current service (BASE, top left), minimization of total costs (TC) scenarios with symmetric headways (TC_SYM, top right), asymmetric headways (TC_ASYM, bottom left), and with mixed vehicle capacities (TC_VEHCAP, bottom left). The thickness of each link is proportional to the joint frequency (over all lines) and the colour indicates the total link capacity in terms of maximum numbers of passengers transportable per hour. In the morning peak, overall network capacity is highest for the ASYM solution which significantly differs from the current supply setting. It is, moreover, remarkable that the link capacities significantly decrease for the VEHCAP scenario. The employment of minibuses seems to be a beneficial option on some lines and, although line frequencies are increased, the reduction in vehicle capacity causes overall supply capacity to decrease.
Figure 5 shows the demand distribution over the network for different supply settings in the morning peak period. The thickness of a link in Figure 5 is proportional to the magnitude of passenger flow between two successive stops. Note that the flows are the results of the dynamic passenger assignment averaged over multiple simulation runs. Different colours indicate the average level of seat occupancy which is computed by the ratio of the load to the total hourly seat capacity of a line segment which is the product of the number of seats of the employed vehicle and the line’s frequency. Since supply is different in each scenario, the resulting levels of vehicle utilization significantly differ. The supply setting resulting from the SYM and ASYM scenarios significantly reduces the average occupancy of vehicles since total supply is increased compared to the base case without changing the vehicles’ capacities. The VEHCAP supply setting, however, significantly increases vehicle utilization on some line segments since link capacities are reduced due to the deployment of smaller vehicles.
4 Conclusion

A tactical decision model for frequency and vehicle capacity determination in public transport networks considering the dynamic interaction between demand and supply is proposed. Results indicate that the current situation in the regarded network can be improved by changing the supply provision in terms of frequencies and vehicle capacities. The use of asymmetric frequency settings can lead to a more effective satisfaction of the present demand and the deployment of multiple vehicle types can even create higher user benefits. The modelling framework can also be applied to strategic network design and tactical supply determination that caters for seasonal trends and special events. Future research should examine the integration of a vehicle scheduling model and a demand elasticity function into the present modelling framework.

References