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System Headways in Line Planning

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Abstract Line Planning is an important stage in public transport planning. This stage determines which lines should be operated with which frequencies. Several integer programming models provide solutions for the line planing problem. However, when solving real-world instances, integer optimization often falls short since it neglects objectives that are hard to measure, e.g., memorability of the system. Adaptions to known line planning models are hence necessary.

We analyze one such adaption, namely that the frequencies of all lines should be multiples of a fixed *system headway*. This is common in practice and improves memorability and practicality of the designed line plan. We model the requirement of such a common system headway as an integer program and compare line plans with and without this new requirement theoretically by investigating worst case bounds, as well as experimentally on artificial and close to real-world instances.

Keywords Public Transport Planning · Line Planning · Integer Optimization

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1 Introduction

Line planning in public transport is a well researched problem. Its goal is to choose the number and the shape of the lines to be operated and to determine their frequencies, i.e., how often services should be offered along every line within the planning period T. The lines together with their frequencies are called a *line concept*. Existing models optimize the costs, e.g., (Claessens et al, 1998b), (Goossens et al, 2006), the number of direct travelers, e.g., (Dienst, 1978),(Bussieck, 1998), or the approximated passengers' traveling times, e.g., (Schmidt, 2014), (Schöbel and Scholl, 2006) of the line concept. Overviews on different models can be found in (Schöbel, 2012) and (Kepaptsoglou and Karlaftis, 2009).

Recent developments include different planning stages into the line planning problems, i.e., they consider integrated planning in public transport. Examples are to integrate the timetabling step (Burggraeve et al, 2017), the demand (Viggiano, 2017) or treating several planning stages in an integrated way (Schöbel, 2017; Huang et al, 2018). Other work examine the effect of time dependent demand (Borndörfer et al, 2018) or the differences of route choice and assignment (Goerigk and Schmidt, 2017).

Nevertheless, solutions to the line planning problem often fall short in important criteria that are not easily measurable in integer optimization problems. One important criterion is the memorability of the resulting timetable. Ideally, public transport passengers need to memorize only one specific minute and a headway for a particular stop, e.g., minute 01 every 10 minutes. To achieve such properties, transport planners use specific concepts when designing line plans. One common concept is a system or pulse headway describing a minimum headway, which must be achieved by all lines, see (Vuchic et al, 1981) and (Vuchic, 2017). The application of a system headway leads not only to regular departure times but also to regular connections when passengers have to transfer.

More precisely, let a line concept consisting of a set of lines $\mathcal{L}$ and their frequencies f_l for all $l \in \mathcal{L}$ be given. If there exists a natural number $i \neq 1$ which is a common divisor of all frequencies f_l we say that the line concept has a *system headway*.

In this paper, we want to model the concept of a system headway mathematically. In particular, we show how the requirement for a system headway can be added to existing integer optimization models, and we derive properties for general line planning models and for a cost-based formulation.

2 Modeling system headways

Before we introduce our adaptions to the integer programming models, we define formally what the line planning problem is. Let a *public transport network* PTN=(V, E) be given, with nodes V as stations and undirected edges E between them. A *line* l is a path in the PTN. In this paper we assume that a *line pool* $\mathcal{L}$ is given. It contains a (large) set of potential lines from which we want to choose the ones to establish. A *line concept* $(\mathcal{L}, f)$ assigns a frequency $f_l \in \mathbb{N}_0$ to every line l in the given line pool $\mathcal{L}$. (Lines which are not chosen from the pool receive a frequency of zero).

There exist many different models for line planning. The frequencies f_l for all $l \in \mathcal{L}$ are the variables to be determined in all line planning models. Sometimes, additional variables $x \in X \subseteq \mathbb{R}^n$ are also present which might for example be used for modeling the paths of the passengers.

The *general line planning model* can hence be written as

$$
\begin{aligned}
\text{(P)} \quad \min \; & obj(f, x) \\
\text{s.t.} \quad & g(f, x) \leq b \\
& f_l \in \mathbb{N}_0 \quad \text{for all } l \in \mathcal{L} \\
& x \in X,
\end{aligned}
$$

where $g : \mathcal{L} \times X \to \mathbb{R}^m$ is a linear function containing m constraints and $b \in \mathbb{R}^m$. Common choices for the linear objective function $obj : \mathcal{L} \times X \to \mathbb{R}$ are to minimize the costs or the traveling time of the passengers, or to maximize the number of direct travelers. The constraints are written in the general form $g(f, x) \leq b$, but as noted in (Schöbel, 2012) most line planning models contain constraints of the type

$$\sum_{\substack{l \in \mathcal{L}: \\ e \in l}} f_l \geq f_e^{\min} \quad \forall e \in E, \tag{LEF}$$

and of the type

$$\sum_{\substack{l \in \mathcal{L}: \\ e \in l}} f_l \leq f_e^{\max} \quad \forall e \in E \tag{UEF}$$

for given lower and upper edge frequency bounds $f_e^{\min} \leq f_e^{\max}$ for every edge $e \in E$. The constraints (LEF) are called *lower edge frequency constraints* and are used to ensure that all passengers can be transported while the *upper edge frequency constraints* (UEF) are needed due to the limited capacity of tracks, or due to noise restrictions. They also bound the costs of the line concept. Allowing to set $f_e^{\min} = 0$ and $f_e^{\max} = \infty$ we can without loss of generality assume that constraints of type (LEF) and (UEF) always are present in the general line planning model.

Typically, *cost-oriented models* minimize the costs of a line concept and contain (LEF) while *passenger-oriented models* optimize the traveling time or the number of transfers passengers have. To prevent the model to establish all lines with high frequencies, constraints of type (UEF) may be used or a budget constraint (BUD) (see Section 5).

The main definition for this work is the following.

Definition 1 A *system headway* (also called *system frequency*) is defined as a common divisor of all frequencies f_l, $l \in \mathcal{L}$, i.e., $i \in \mathbb{N}$ is a system headway for $(\mathcal{L}, f)$ if and only if $i \geq 2$ and $i|f_l$ for all $l \in \mathcal{L}$.

In the following we look for line concepts which have a system headway. Note that we only consider system headways greater than one, as choosing $i = 1$ as a system headway poses no restriction on the model and is therefore considered as having no system headway at all.

Including the system headway requirement into the general line planning model (P) is possible with only small adaptions. Let us first consider a *given and fixed* system headway $i \in \mathbb{N}$. Since the frequencies f_l are integer variables we can include a system headway by adding only the constraints (1) and (2):

$$
\begin{aligned}
(\mathrm{P}(i)) \quad & \min \ obj(f, x) \\
& \text{s.t.} \quad g(f, x) \leq b \\
& \qquad f_l = \alpha_l \cdot i \quad \forall l \in \mathcal{L}, && (1) \\
& \qquad \alpha_l \in \mathbb{N}_0 \quad \forall l \in \mathcal{L} && (2) \\
& \qquad f_l \in \mathbb{N}_0 \ \text{ for all } l \in \mathcal{L} \\
& \qquad x \in X.
\end{aligned}
$$

By opt(i) we denote the optimal objective function value of P(i). At first it is unclear, whether (1) and (2) add to the difficulty of the model. In fact, they do not do this, as the following theorem shows.

Theorem 1 *Let (P) be a general line planning problem for a given instance based on the period T. Then problem P(i) is equivalent to a line planning problem (P'). The new line planning problem (P') has the same number of variables and constraints as (P).*

Proof We introduce new variables $f_l' := \frac{f_l}{i}$ for all $l \in \mathcal{L}$. Substituting f_l by these new variables in P(i) and using the linearity of *obj* and of g, we receive

$$
\begin{aligned}
(\mathrm{P'}(i)) \quad & \min \ i \cdot obj(f', x) \\
& \text{s.t.} \quad i \cdot g(f', x) \leq b \\
& \qquad i \cdot f_l' = \alpha_l \cdot i \quad \forall l \in \mathcal{L} \\
& \qquad \alpha_l \in \mathbb{N}_0 \quad \forall l \in \mathcal{L} \\
& \qquad f_l \in \mathbb{N}_0 \ \text{ for all } l \in \mathcal{L} \\
& \qquad x \in X.
\end{aligned}
$$

From $i \cdot f_l' = i\alpha_l$ we conclude that $f_l = \alpha_l$ for all $l \in \mathcal{L}$ and the variables α_l are not needed any more. P'(i) hence simplifies to

$$
\begin{aligned}
(\mathrm{P'}(i)) \quad & \min \ obj(f', x) \\
& \text{s.t.} \quad g(f', x) \leq \tfrac{b}{i} \\
& \qquad f_l \in \mathbb{N}_0 \ \text{ for all } l \in \mathcal{L} \\
& \qquad x \in X.
\end{aligned}
$$

which is a line planning problem with the same number of variables and constraints, but a right hand side $\frac{b}{i}$. $\square$

Note that the new line planning problem can be interpreted as using the period $T' := \frac{T}{i}$ instead of T. This can be seen by looking at (LEF) and (UEF) which in (P') now read as

$$\frac{f_e^{\min}}{i} \leq \sum_{\substack{l \in \mathcal{L}: \\ e \in l}} f_l \leq \frac{f_e^{\max}}{i} \quad \forall e \in E,$$

i.e., we restrict how many vehicles are allowed to pass an edge in the new period $T' := \frac{T}{i}$.

Example 1 We are interested in a solution with system headway $i = 4$. Then instead of using lower and upper edge frequency bounds of 3 and 6, respectively, we can bound the number of vehicles running along this edge within 15 minutes to be between $\frac{3}{4}$ and $\frac{6}{4}$. Since

$$\sum_{\substack{l \in \mathcal{L}: \\ e \in l}} f_l \in \mathbb{N}$$

we can furthermore use integer rounding and obtain the only feasible solution of four vehicles per hour running along this particular edge.

It might also be interesting to determine the line concept with a *best possible* system headway, i.e., we have no particular number i for a system headway given but we wish to find a line concept which satisfies the system headway requirement for some natural number $i \geq 2$. A naive approach is to solve $P(i)$ for all i smaller than the period length T and choose the solution with best objective value opt(i). However, choosing the best possible system headway can also be formulated as an integer quadratic program by adding the constraints (3) and (4) to P(i) and hence leaving $\alpha = i$ as variable:

$$\begin{aligned}
(\mathrm{P}_{sys-head}) \quad \min \;\; & obj(f,x) \\
\text{s.t.} \quad & g(f,x) \leq b \\
& f_l = \alpha_l \cdot \alpha \quad \forall l \in \mathcal{L}, \\
& \alpha_l \in \mathbb{N}_0 \quad \forall l \in \mathcal{L} \\
& \alpha \geq 2 && (3) \\
& f_l \in \mathbb{N}_0 \;\; \text{for all } l \in \mathcal{L} \\
& x \in X \\
& \alpha \in \mathbb{N}. && (4)
\end{aligned}$$

In the following we analyze which system headways are reasonable and how much one loses in quality or costs of a line plan when (the best) system headway is chosen. We first have a look at the general line planning problem and then discuss the classic cost-oriented model and the direct travelers approach.

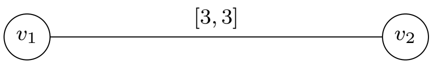


Fig. 1: Infrastructure network for Example 2

3 The size of a system headway in the general line planning problem

In this section we investigate which numbers i are suitable as system headways and how we can find a best solution among all possible system headways.

In the following we compare the result of $P(i)$ for different values of i. Our first result states that a divisor i of a given system headway j always yields a better solution than using j itself. This holds for all general line planning problems.

Lemma 1 *Let $i, j \in \mathbb{Z}$ and $i|j$. Then* $\text{opt}(i) \le \text{opt}(j)$.

Proof Let $(f(i), x(i))$ denote a feasible solution to $P(i)$, and $(f(j), x(j))$ denote a feasible solution to $P(j)$. This means $j|f(j)$. Together with the assumption $i|j$ we obtain that $i|f(j)$, hence $f(j)$ satisfies (1) and (2) also in $P(i)$. The other constraints $g(x, f) \le b$ of $P(i)$ are also constraints of $P(j)$, hence every feasible solution for $P(j)$ is also feasible for $P(i)$ and their objective functions coincide. Therefore, $P(i)$ is a relaxation of $P(j)$ and $\text{opt}(i) \le \text{opt}(j)$. □

The previous lemma shows that searching for the best solution using a system headway can be done more efficiently: Instead of testing every possible value, it is enough to restrict ourselves to prime numbers.

Corollary 1 *There always exists an optimal solution (α, f, x) to $(P_{sys-head})$ in which the optimal system headway α is a prime number.*

Unfortunately, it cannot be seen beforehand which prime number results in the best solution. In practice, choosing a smaller system headway is often better (as can be seen in Section 6). However, depending on the constraints $g(f, x) \le b$, there are counterexamples where a smaller system headway is not even feasible. This is even true if $g(f, x) \le 0$ only consists of lower and upper edge frequency constraints (LEF) and (UEF) as the following example shows.

Example 2 Consider a simple PTN with only two stations and a connecting edge, as depicted in Fig. 1. Let the lower and upper edge frequencies of this edge be both set to three. Then there is a feasible solution for a system headway of $i = 3$ but not for $i = 2$.

Such examples raise the question in which cases $(\text{P}_{sys-head})$ has a feasible solution. Clearly, if the original line planning problem (P) is infeasible then certainly also all P(i) and $(\text{P}_{sys-head})$ are. As Example 2 shows, (LEF) and (UEF) already make the opposite direction of this statement wrong: P(i) can be infeasible even if (P) is feasible. The next lemma shows that this happens in particular for small upper edge frequencies $f_e^{\max}$:

Lemma 2 *Let (P) be a general line planning problem containing constraints of type* (LEF) *and of type* (UEF)*. ($P_{sys-head}$) is infeasible if there exists an edge e with $f_e^{\min} = f_e^{\max} = 1$.*

Proof Edge e needs to be covered by exactly one line l with frequency $f_l = 1$ which then is not an integer multiple of any $i \geq 2$. □

On the other hand, in case the only constraints contained in $g(l,x) \leq b$ are constraints of type (LEF), then we have a positive result.

Lemma 3 *Let (P) be a feasible line planning problem in which only has constraints of type* (LEF) *or constraints which depend on x, but not on f. Then P(i) is feasible for all possible system headways $i \geq 2$.*

Proof Take a solution (f,x) for (P). For all $l \in \mathcal{L}$ define

$$f'_l := \min\{k : i|k \text{ and } k \geq f_l\}.$$

Then f'_l satisfies (1) and (2). Furthermore, since $f'_l \geq f_l$ also (LEF) are satisfied, and satisfaction of constraints which just depend on x is not changed when replacing f by f'. Hence, (f',x) is a feasible solution to P(i). □

Note, that even if the conditions of Lemma 3 are met, a smaller system headway does not need to be better, as can be seen in Example 3.

4 Bounds for a cost model in line planning

We now turn our attention to a particular model in line planning, namely the *basic cost model*. It has been extracted from the cost model in Claessens et al (1998a) and stated in Schöbel (2012). The model allows to study how much we lose when requiring a system headway compared to the original model without the system headway requirement.

Since we know from Lemma 2 that (UEF) may destroy feasibility of line planning problems we only consider problems without upper edge frequency bounds for the rest of this section, i.e.,

$$f_e^{\max} = \infty \quad \forall e \in E.$$

The cost model we study here is the following: Passengers are first routed along shortest paths in the PTN. The number of passengers which travel along edge e in these shortest paths is then counted and divided by the (common) capacity of the vehicles. This gives the minimal number of vehicles $f_e^{\min}$ needed to cover edge e The costs of a line concept are approximated as

$$cost(\mathcal{L}, f) = \sum_{l \in \mathcal{L}} f_l \cdot cost_l,$$

where $cost_l$ is a given cost per line $l \in \mathcal{L}$. This often includes time- and distance-based costs of a line. In this work, we pose no assumptions on the structure of the costs $cost_l$, i.e., they can be chosen arbitrarily for each line. Including the system headway requirement results in model P(i):

$$\begin{array}{lllll} \min & & \sum\limits_{l \in \mathcal{L}} f_l \cdot cost_l & & \\ \text{s.t.} & f_e^{\min} & \leq & \sum\limits_{\substack{l \in \mathcal{L}: \\ e \in l}} f_l & \forall e \in E \\ & f_e^{\max} & \geq & \sum\limits_{\substack{l \in \mathcal{L}: \\ e \in l}} f_l & \forall e \in E \\ & f_l & = & \alpha_l \cdot i & \forall l \in \mathcal{L} \\ & f_l, \alpha_l & \in & \mathbb{N}_0 & \forall l \in \mathcal{L} \end{array} \tag{P(i)}$$

As before, opt(i) denotes the optimal cost value for $P(i)$.

First note, that even in this simple model, $opt(i) \leq opt(j)$ for $i \leq j$ need not hold as the next example shows.

Example 3 Consider again the simple PTN of Fig. 1. Let the lower edge frequency of this edge be three as before, while the upper edge frequency is now deleted (or set to $f_e^{\max} = \infty$). Let only one line l serve edge e. Then the optimal solution for a system headway of $i = 3$ is $f_l = 3$ which leads to an objective function value $opt(3) = 3 \cdot cost_l$. Now, taking a smaller system headway of $i = 2$ requires a frequency of $f_l = 4$ for line l in order to serve edge e. This means we obtain

$$opt(2) = 4 \cdot cost_l > 3 \cdot cost_l = opt(3).$$

Nevertheless, even if monotonicity does not hold, the structure of the cost model allows to prove the following result.

Theorem 2 *Let* $i, j \in \mathbb{Z}$, $i \leq j$. *Then* $\text{opt}(j) \leq \frac{j}{i}\text{opt}(i)$.

Proof Let f^i be an optimal solution to $P(i)$. Then $f' = \frac{j}{i} f^i$ is a feasible solution for $P(j)$, since $j | f'$ and the lower edge frequency requirements (LEF) are still satisfied:

$$\sum_{\substack{l \in \mathcal{L}: \\ e \in l}} f'_l = \sum_{\substack{l \in \mathcal{L}: \\ e \in l}} \frac{j}{i} f_l^i \geq \sum_{\substack{l \in \mathcal{L}: \\ e \in l}} f_l^i \geq f_e^{\min} \quad \forall e \in E.$$

Therefore, the optimal objective value of $P(j)$ can be bounded by the objective value of f' :

$$\mathrm{opt}(j) \leq \sum_{l \in \mathcal{L}} f'_l \cdot cost_l = \sum_{l \in \mathcal{L}} \frac{j}{i} f^i_l \cdot cost_l = \frac{j}{i} opt(i).$$

□

Note that this lemma also holds for $i = 1$, i.e., the case for no system headway. This yields the following corollary.

The result also allows to compare the costs of an optimal solution for the original problem (P) to the costs of an optimal solution for problem $P(i)$ with a system headway of i.

Corollary 2 *Let* opt *be the optimal objective value of the cost model. Then the optimal costs* $\mathrm{opt}(i)$ *of a system headway i compared to the model without the requirement of a system headway are bounded by*

$$\mathrm{opt}(i) \leq i \cdot \mathrm{opt}^*.$$

Therefore requiring a system headway of, e.g., $i = 2$ can in the worst case double the costs.

Although this factor is often not attained in practice (see Section 6), the bound is sharp.

Example 4 Consider again the simple PTN of Fig. 1 but now with a lower edge frequency of one, i.e., the edge must be covered and only one line l serving edge e. Then the optimal solutions for a system headway of 2 and 3 fulfill:

$$opt(2) = 2 \cdot cost_l = \frac{2}{3} \cdot 3 \cdot cost_l = \frac{2}{3} opt(3)$$

5 Passenger-oriented models

There are several passenger oriented models known in literature. We mainly consider the direct traveler model introduced in (Bussieck, 1998). For this problem, the number of direct travelers, i.e., the number of passengers that can travel from their origins to their destinations without changing lines, should be maximized. Other models try to minimize the approximated travel time of the passengers, e.g., (Schöbel and Scholl, 2006; Borndörfer et al, 2007).

Passenger oriented models need other types types of constraints than those in the cost model of Section 4. Including (LEF) may not be necessary any more since the passengers are treated in the objective function. Including (LEF) is one way to restrict the costs of the line plan (and used, e.g., in Bussieck (1998)). There may also be a budget constraint in the form of

$$\sum_{l \in \mathcal{L}} cost_l \cdot f_l \leq B, \tag{BUD}$$

where $cost_l$ are given cost coefficients for every line $l \in \mathcal{L}$ which may include time- and distance-based costs of a line. In this work, we pose no assumptions on the structure of the costs $cost_l$, i.e., they can be chosen arbitrarily for each line.

When we remove such a constraint from a passenger oriented model, the problem often becomes trivial, since it might be an optimal solution to establish all lines with high frequencies (which can then be chosen as multiples of the given system headway i). Hence, a constraint of the type of (BUD) is necessary. However, with a budget constraint, we obtain similar problems to Lemma 2, as can be seen in the following example.

Example 5 We again consider the PTN given in Fig. 1. When we now assume that we have a budget constraint restricting the costs of the solution to a single line with frequency 1, there is no feasible solution for any system headway.

Similarly, we can construct examples equivalent to Example 2 and Example 3.

The conclusion is the following: It can always happen that the original line planning model (P) is feasible while the corresponding problem P(i) with a fixed system headway i or even ($\mathrm{P}_{sys-head}$) become infeasible. This means that a result such as Theorem 2 for the cost model is not possible for (reasonable) passenger-oriented models and that the relative difference between the objective of a system headway and the objective without this requirement may be arbitrarily large.

6 Experiments

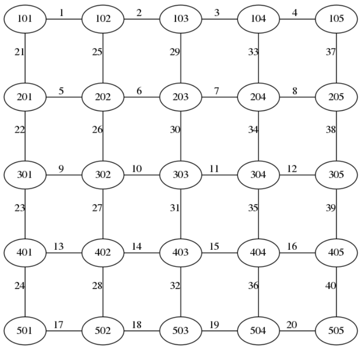
For the practical experiments, we consider three instances with different characteristics:

`Grid`: A small example first presented in (Friedrich et al, 2017). It is designed to be small enough to understand effects of decisions but still contains a realistic demand structure. It has 25 stops, 40 edges and 2546 passengers. For a representation of the infrastructure, see Fig. 2a. The instance has been tackled by several researchers and can be downloaded at (Grid, 2018).

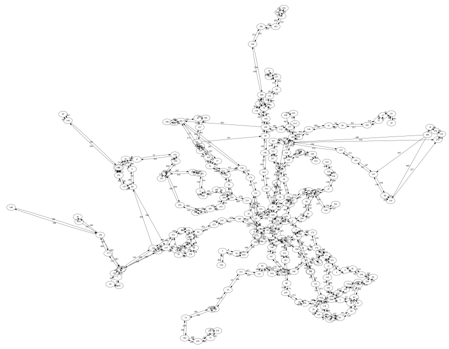
`Goettingen`: An instance based on the bus network in Göttingen, a small city in the geographical center of Germany. It contains 257 stops, 548 edges and 406146 passengers. For a representation of the infrastructure, see Fig. 2b.

`Germany`: An instance based on the long-distance rail system in Germany. It contains 250 stops, 326 edges and 3147382 passengers. For a representation of the infrastructure, see Fig. 2c.

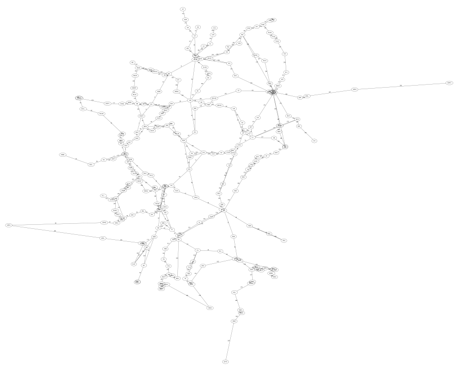
All experiments are done using the `LinTim`-software framework (Goerigk et al, 2013; Schiewe et al, 2018). We computed a line concept without system headway as well as for every system headway from 2 to 10 while optimizing the given line planning problem.



(a) `Grid`



(b) `Goettingen`



(c) `Germany`

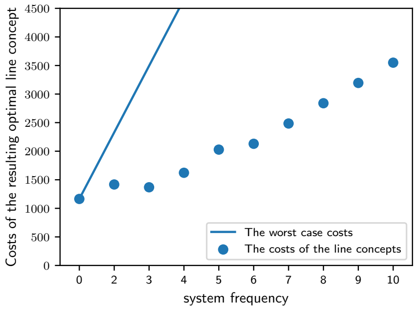
Fig. 2: Infrastructure networks of the used instances

First, we consider solving the cost model discussed in Section 4. An evaluation containing the costs of the different solutions and the worst case costs of Lemma 2 can be found in Fig. 3.

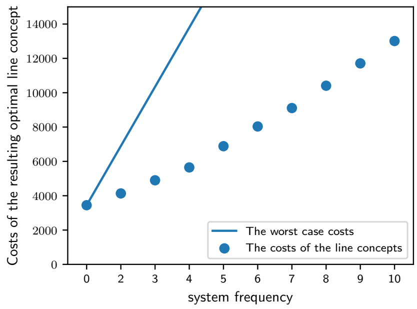
There are mainly two things to observe here: First of all, the assumption that higher system headways lead to higher costs is often, but not always true. In all but one case, the costs are strictly increasing for increasing system headways.

But, as was seen in Section 3, this does not always have to be the case. This can be observed in Fig. 3a where the solution for a system headway of $i = 3$ has lower costs than the solution for a system headway of $i = 2$. This occurs in cases where the demand on most edges can be met by lines with a frequencies of three. Then a system headway of $i = 2$ leads either to more lines or to line frequencies of four.

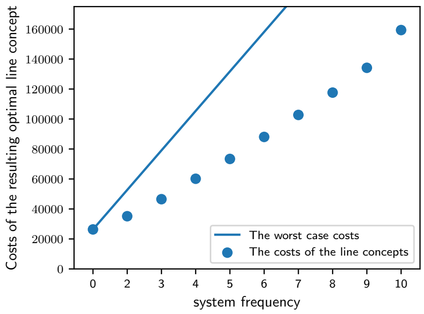
Additionally, note that the worst case factor for using a system headway from Lemma 2 is not obtained in practice but the difference to the theoretical bound decreases with increasing instance size.



(a) `Grid`

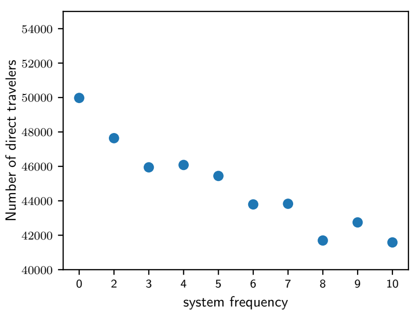


(b) `Goettingen`

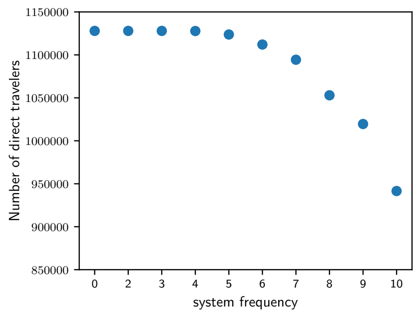


(c) `Germany`

Fig. 3: Solutions for the Cost Model

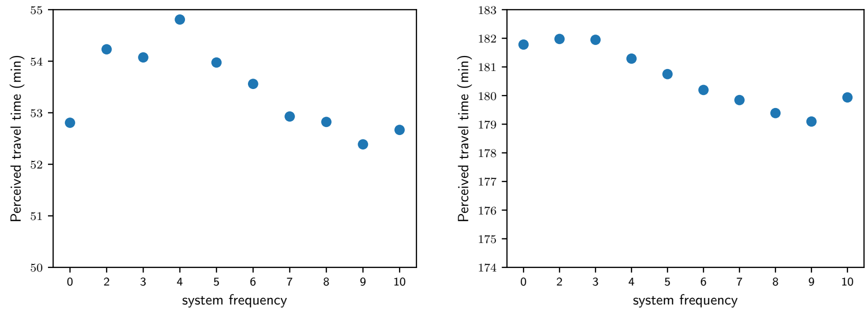


(a) `Goettingen`



(b) `Germany`

Fig. 4: Solutions for the Direct Travelers Model



(a) `Goettingen` (b) `Germany`

Fig. 5: Evaluation of the Timetables

Next, we consider the case of a passenger-oriented line planning model. We chose the direct travelers model of (Bussieck, 1998), see also Section 5. For this, we set a budget to examine the effect of the system headway on a restricted problem.

In Fig. 4 we can clearly see the effects of the system headway.

In the instance `Goettingen` (Fig. 4a), we again observe that the quality of the line plan decreases most of the times with increasing system headway but there may be cases where a bigger system headway can use the given budget a little bit better, resulting in a better plan for the passenger. Hence, monotonicity of the objective function is also here likely, but not guaranteed.

In the instance `Germany` (Fig. 4b), we see the effect of a late drop-off of the quality, resulting from a budget that is big enough to not be restrictive for the first few cases.

It has been recognized in several publications (Burggraeve et al, 2017; Schöbel, 2017; Huang et al, 2018) that line planning should not be treated isolated from other planning stages, but an integrated approach is needed. We are hence interested not only in the effects a system headway has on line plans, but also consider if there are effects on the resulting timetable. Note that the line plan influences the resulting passengers' travel time obtained by the timetable significantly Friedrich et al (2017); Goerigk et al (2013).

To consider the results of system headways on the timetable, we compute a periodic timetable for each of the line plans and compare their qualities, evaluating the *perceived travel time* of the passengers in the timetable, i.e., the travel time including a small penalty for every transfer. For the computation of the timetable, we use the fast *MATCH* approach introduced in (Pätzold and Schöbel, 2016). The results are depicted in Fig. 5.

Again, we see the anticipated results: A higher system headway results in a public transport supply with shorter headways. This leads in many cases to shorter transfer waiting times and reductions in the perceived travel time, indicating a higher quality for the passengers. However, also here, this interrelation does not apply without exception as Fig. 5 shows.

7 Outlook

We added the system headway constraint to line planning models, derived theoretical bounds on their effects and examined the results on practical instances for a cost model and a passenger-oriented model. It would be interesting to see the proposed system headway adjustments implemented into even more line planning models to further extend the comparison and examine the effects on public transport systems.

Another interesting topic is the evaluation of the impact of a system headway on passengers. Important metrics, such as the memorability of a timetable, can only be measured inadequately using the state-of-the-art mathematical evaluation systems and can therefore not be compared conclusively. One way of evaluating the impacts is to estimate the changes in public transport travel demand. This requires a mode choice model, which captures not only travel time and number of transfers as indicators for service quality, but also the service frequency and the regularity. This can be achieved by an indicator adaption time, which quantifies the time difference between the desired departure time of a traveler and the provided departure time of the public transport supply. In car transport the adaption time is always zero. A public transport supply with regular and short headways reduces adaption time and thus makes public transport more competitive. Experiments with the grid instance indicate that especially in networks with low demand the additional costs of a system headway can partially be compensated by a shift from car to public transport. In networks where high demand leads to solutions with headways below 10 minutes, the impact of a system headway on additional cost and demand is smaller. Here the modal share primarily depends on differences in in travel time and travel costs. Future work is necessary to better understand the impact of regularity and adaption time on passengers travel behavior.

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