An Optimization Model for Planning Limited-Stop Bus Operations

Mahmood Mahmoodi Nesheli · Siva Srikukenthiran · Amer Shalaby

Abstract Surface transit lines in North America commonly feature a basic service pattern consisting of a single branch of all-stop service, with stops usually tightly spaced. Such configuration is both inefficient to the operator and unattractive to the users, particularly if the prevailing passenger demand is unevenly distributed along the line. In such cases, it is more effective to tailor the scheduled services to passenger demand, both spatially and temporally. Public Transit agencies have increasingly adopted various stop and service pattern strategies in order to provide high quality services while reducing operating costs. This study focuses on one such strategy, namely limited stop operation. It proposes a new mathematical programming model to find the best candidate route stops for this strategy so as to minimize the total passenger travel time. The adopted approach consists of two steps: optimization and simulation. An agent-based simulation platform, called Nexus, is used to represent real-life operating conditions, generate input data for the optimization model, and finally analyze and assess the optimization results and present the optimal strategy. The developed approach is tested in a case study of a transit system in Hamilton, Ontario, Canada. Multiple analysis and algorithm test cases are demonstrated.
Keywords Public transport · Stop service pattern · Limited stop strategy · Agent-based simulation.

1 Introduction

The design of stop and service patterns (SSP), a critical task in transit planning, can have remarkable impacts on user travel times and system efficiency (Vuchic, 2005). Stop and service patterns include several specialized service strategies designed to expedite operations and tailor service to unevenly distributed passenger demand along a public transit route; examples include stop consolidation, limited stop service, zonal service and short turn routes. Agencies may consider one strategy at a time or a few in combination. For example, a line could be re-designed by consolidating some stops or introducing a limited-stop branch in conjunction with a basic all-stop branch. These decisions on SSP need to be supported by appropriate analytical tools and informed by the detailed data on both demand and performance enabled by automated data collection systems, such as Automatic Fare Collection (AFC), Automatic Passenger Counting (APC) and Automatic Vehicle Location (AVL).

In this study, one specialized service strategy, the limited-stop strategy, is investigated to examine its potential to better align the provided service with demand. We introduce a new model to select the best candidate route and stops for this strategy so as to minimize the total passenger travel time.

2 Literature review

Several studies have discussed different strategies and approaches to address the inefficiency of surface transit system. Generally speaking, control strategies can be categorized into three main categories: stop control, inter-stop control, and others (Eberlein et al, 1999). The first category includes two main classes of strategies which are known as holding and stop skipping/limited-stop. The second includes speed control and traffic signal priority. The third consists of strategies such as adding vehicles, splitting trains, and more.

Early research into such strategies includes the zonal operation studies by Jordan and Turnquist (1979) and the bi-directional extension of zonal service optimization by Furth (1986). Eberlein (1995) addressed the limited-stop problem as an integer nonlinear programming with quadratic objective function, and solved it with a heuristic method. The optimization of express operations is examined by Leiva et al (2010) and Ulusoy et al (2010) showing that these strategies can yield significant time savings and social benefits. The analysis of grid and non-grid patterns for coordinated express services is performed by Wirasinghe and Vandebona (2011). Social welfare benefits of limited-stop services, when coordinated with regular services making all stops, are investigated by Chiraphadhanakul and Barnhart (2013). Ttrenault and El-Geneidy (2010) used AVL and APC data to select stops and estimate running times for
the limited-stop service. Their finding shows that the usage of limited-stop service would result in substantial travel time savings for both limited-stop and existing local services. Chen et al (2015) proposed a mathematical model for the optimal stopping design of limited-stop bus service. The study considers the vehicle capacity and stochastic travel time and stops are not allowed to be skipped by two consecutive vehicles. Soto et al (2017) developed a bi-level optimization method to design limited-stop services by considering bus capacity, transfers, and different types of behavioral models for the passengers.

Concerning operational control strategies, Nesheli and Ceder (2015) investigated how hybrid special operations, including skip-stop strategies, could be used for transfer synchronization.

The previous efforts mainly focused either on developing complicated optimization methods which were difficult to implement in the practice, or used some simplified analytical models to address the effects of any given SSP configuration. In contrast, this study utilizes a route simulation model to provide more realistic metrics for the optimization model. As such, the objective of this work is to develop a framework combining an optimization model and a simulation model for determining the best candidate stops for a limited stop strategy at the planning stage.

3 Model development

The introduction of a limited-stop (LS) strategy is beneficial to consider for high-frequency routes, where high-demand stops are served by all route vehicles while stops with low demand are skipped by some vehicles. The benefits of the LS strategy mainly accrue to "through" passengers aboard a vehicle skipping a given stop and those who will board at a downstream stop served by the vehicle. However, this strategy has an adverse effect on passengers who want to board at the skipped stop since they have to wait for the next available vehicle or walk to the adjacent stop. In this section we introduce an optimization model to develop an optimum bus service operation for a given bus route. The proposed model seeks to modify the bus timetable by optimally reassigning some local trips to operate LS service. Therefore, the LS service operates in parallel to the local service to minimize the total passenger travel time.

3.1 Assumptions

Usually, the LS strategy is constrained by the passengers who want to alight the vehicle at a stop planned to be skipped. It is assumed that passengers aboard a vehicle are aware of a vehicle’s specific service pattern before boarding. It is also assumed that any LS introduction does not modify rider behavior in terms of Mode choice and Route choice. The model assumes the availability of O-D demand matrices for routes under consideration.
3.2 Model formulation

Consider a single-line public transit (PT) corridor with $N = \{1, 2, \ldots, |N|\}$ stops. A route is made up of a collection of "trips" in each direction. Each trip $k$ represents a single vehicle run, with specific departure times at a series of stops along the route. The variables used in the model formulation are defined as follows:

- $N$: set of stops, with $n \in N$;
- $K$: set of vehicle trips, with $k \in K$;
- $b_{k,n}$: number of boarding passengers at stop $n$ for trip $k$;
- $a_{k,n}$: number of alighting passengers at stop $n$ for trip $k$;
- $l_{k,n}$: number of passenger load on trip $k$ at stop $n$;
- $c_{k,n}$: vehicle running time on trip $k$ at stop $n$ which is defined as the vehicle running time between stops $n - 1$ and $n$;
- $d_{k,n}$: vehicle dwell time on trip $k$ at stop $n$;
- $H_k$: vehicle headway time on trip $k$;
- $S_{k,n}$: binary decision variable, 1 if trip $k$ at stop $n$ is skipped, otherwise 0;

While the benefit of the LS strategy is the in-vehicle time saved for through passengers, it increases the travel time for passengers who want to alight or board at the skipped stops. To consider the impact of the LS strategy on the waiting times of different passenger groups the following terms are defined.

Let $\Theta_{k,n} = \sum_{i=n+1}^{N} b_{k,i}$, be the the total number of passengers boarding trip $k$ downstream of stop $n$. Clearly, the expected wait time for passengers that are waiting for the bus downstream of stop $n$ is affected by applying a LS strategy, where stop $n$ is skipped. These passengers will experience shorter wait times.

On the other hand, the passengers’ travel time will change because of using the LS strategy at the previous stops (i.e., before stop $n$). Let $\Lambda_{k,n} = \sum_{i=1}^{n-1} (S_{k,i}d_{k,i})$, be the time change due to the introduction of LS strategy at the upstream stops. That is a bus will arrive to stop $n$ (to be a skipped stop) in a shorter time. As a result, the time change will be added to the planned headway and increases the time interval for the next service to visit this stop. Therefore, $\Lambda_{k,n}$, is an additional wait time for the out-of-vehicle passengers of trip $k$ at stop $n$ due to the applied LS strategy at upstream stops, resulting in longer wait time.

It is also important to consider those passengers who wanted to board the previous service ($k - 1$) but because of using the LS strategy at stop $n$ they need to take the next available trip. Let $\Gamma_{k,n} = S_{k-1,n}b_{k-1,n}$, be the number of extra boarding passengers on trip $k$ at stop $n$ due to the applied strategy to the previous trip. These passengers will experience longer waiting times and will board the next available vehicle.

One of the challenges in the use of any strategy is evaluating how these strategies affect the passengers’ travel experiences between their origin and destination. Recent advancement in data and information technology; such
as smart card data, have made it possible to obtain highly disaggregate O-D passenger demand (stop to stop) for each route. This O-D information has been considered in the formulation as follows. First, O-D matrix passenger-travel time (PTOD) for each trip is generated from O-D matrix demand, and O-D matrix travel time. Then a ratio on the basis of those passengers who are affected by applying a LS strategy is computed. That is the numerator refers to the O-D matrix of affected passenger-travel time and the denominator refers to the O-D matrix of total passenger-travel time. Let $\Psi_{k,n}^a$ be the defined O-D ratio of trip $k$ at stop $n$. As the following expression shows, the value of $\Psi_{k,n}^a$ depicts the fraction of the travel times by those passengers who wanted to alight at the stop that is not served and should use instead the alternative stops.

$$\Psi_{k,n}^a = \frac{\sum_{i=1}^{n} PTOD_{k,i,n}}{\sum_{i=1}^{[N]-1} \sum_{j=i+1}^{[N]} PTOD_{k,i,j}} \forall i, j \in N$$

Like, alighting passengers, it is possible to consider the impact of O-D passengers travel time effect on those passengers who wanted to board at not-served stops by the following formula:

$$\Psi_{k,n}^b = \frac{\sum_{i=1}^{n} PTOD_{k,n,i}}{\sum_{i=1}^{[N]-1} \sum_{j=i+1}^{[N]} PTOD_{k,i,j}} \forall i, j \in N$$

*Penalty function:* In order to consider the drawback of a LS strategy on the impacted passengers and to examine different alternatives for them, a set of penalty functions are introduced. Figure 1 illustrates these penalties.

a) *Walking time penalty:* Let the "upstream served stop" be the last stop served before the skipped stop, and the "downstream served stop" be the first stop served after. It is possible to categorize two types of penalty functions to take into account the impact of the LS strategy on the disadvantaged passengers.

In first type, passengers who want to alight at the not-served (i.e. skipped) stops but will instead alight at the downstream served stop will need extra time to reach their destination; this is termed forward time penalty ($P^F$), and formulated as follows:

$$P^F_{k,n} = (f - 1) \sum_{i=1}^{n} c_{k,i} \prod_{i=q}^{n} S_{k,i} \forall q, n \in N, \{1 \leq q < n\}$$

where $f$ is the ratio of an average bus speed over average walking speed of a pedestrian. The impacted passengers are from stop $q$ (the stop that passenger decide to alight) to stop $n$ (the stop that is not served by trip $k$). These passengers will not experience bus running time along the segments upstream of the non-served stops.
The second type of penalty, pertaining to passengers who alight at the "downstream served stop", will need to return to their desired stop \( n \), with this additional time given the term backward time penalty \( P^B \), and formulated as follows:

\[
P^B_{k,n} = (f + 1) \sum_{i=n+1}^{N} c_{k,i} \prod_{i=n}^{q} S_{k,i} \quad \forall q, n \in \mathbb{N}, \{n < q < |\mathbb{N}|\} \tag{4}
\]

Note that, these passengers will experience additional bus running time beyond the non-served stop because of staying in the bus and alighting at the next stop.

We thus obtain the walking time penalty as follows:

\[
P^{\text{walk}}_{k,n} = \min(P^F_{k,n}, P^B_{k,n}) \tag{5}
\]

b) Waiting time penalty: The other possibility for passengers, which occurs when the walking distance is relatively long, is to wait for the next bus to reach their desired stops. This wait time could be associated with the effect of LS strategy used on previous trip. Thus,

\[
P^{\text{wait}}_{k,n} = H_k + \sum_{i=1}^{[|N|]} S_{k-1,i} d_{k-1,i} \tag{6}
\]

Using passengers' waiting and walking times to construct the penalty function of those who wanted to use non-served stops, the total alighting penalty time \( P^{\text{alight}} \) takes the form:

\[
P^{\text{alight}}_{k,n} = \min(P^{\text{walk}}_{k,n}, P^{\text{wait}}_{k,n}) \tag{7}
\]
c) Stop-waiting penalty: Similarly, those who wanted to board at the stops that are not served (skipped) should wait for the next vehicle. Therefore, the formulation of penalty time of this group of passengers is associated with the upstream stops, where LS strategies are used, and can be written as follows:

\[ P_{S,wait}^{k,n} = H_k + A_{k,n} \]  

(8)

Based on the above equations, the effect of the limited-stop strategy on the change in the total travel time with respect to trip \( k \) and stop \( n \) can be expressed in equation 9. Note that, the passengers’ wait time components are weighted by \( \delta_1 \) for out-vehicle wait time and \( \delta_2 \) for in-vehicle wait which can be set by the empirical data.

\[ LS_{k,n} = S_{k,n} \left[ (1 + \psi^a_{k,n})a_{k,n} + (1 + \psi^b_{k,n})b_{k,n} + \delta_1 \Gamma_{k,n} (2H_k + A_{k,n}) - \delta_2 d_{k,n} (l_{k,n} + \Theta_{k,n}) \right] \]

(9)

The objective of the model is to minimize the total passenger travel time, which is computed as the total possible increase in out-of-vehicle passengers’ wait time and decrease in in-vehicle passengers’ travel time by the introduction of the LS strategy.

Consequently, the objective function for the proposed model to find the optimal LS strategy can be written as:

\[ \min \sum_{k \in K} \sum_{n \in N} LS_{k,n} \]

(10)

**LS service constraints:** To make sure the LS strategy meets the service requirements under different conditions, a set of constraints need to be considered in the model. The following constraints ensure that the values of LS constitutes a valid service.

\[ S_{k,1} = 0 \]  

(11)

\[ S_{k,1}Z_{k,n} = 0 \quad \forall Z \in \{0, 1\} \]

(12)

where \( Z_{k,n} \) is 1 when \( n \) is a special stop that cannot be skipped, otherwise zero.

\[ S_{k,n}Y_{k,n} = 0 \quad \forall Y \in \{0, 1\} \]

(13)

where \( Y_{k,n} \) is 1 when \( n \) is a transfer stop that cannot be skipped, otherwise zero.

\[ S_{k,n} + S_{k,n+1} \leq 1 \]

(14)

\[ S_{k,n} + S_{k+1,n} \leq 1 \]

(15)

Constraint 11 ensures that the first stop is served. Constraints 12 and 13 guarantee that important stops such as transferring stops and some special
stops with amenities cannot be skipped. Constraint 14 ensures that two consecutive stops cannot be skipped. Constraint 15 ensures that the LS strategy cannot be applied to consecutive trips at the same stop.

To check that the bus capacity meets the available demand when the bus departs the stop the following constraint can be added. The value of $\nu$ indicates the capacity of a bus.

$$lk,n + bk,n - ak,n < \nu^{max}$$

(16)

3.3 Solution approach

The developed model is an integer problem with a complexity of $(k \times n)$. The problem is formulated to be solved using a combination of relaxations (strengthened by cutting-planes) and "branch and bound" techniques. In addition, a constrained programming (CP) technique is used. The CP approach makes decisions on variables and values and, after each decision, performs a set of logical inferences to reduce the available options for the remaining variables’ domains. Thus, the CP allows us to efficiently address the LS service problem for a real-world PT system.

3.4 Simulation analysis

As the limited-stop service design optimization process described in this paper was developed to harness the emerging wealth of sensor data produced by modern-day transit systems, the method expects detailed input data on the movement of both transit users and service vehicles. It also requires a method that takes into account the complex dynamics of transit actors to properly evaluate the impact of any proposed service changes. With traditional transit demand data sources and service modelling methods proving insufficient, particularly for vehicle trip-level O-D, this study utilizes mesoscopic simulation, namely the Nexus simulation platform previously developed at the University of Toronto (Srikukenthiran and Shalaby, 2017).

Nexus is an agent-based simulator with integrated transit assignment, allowing for detailed metrics on both service and user experience. Initially developed to enable highly detailed simulation of large-scale regional multi-modal networks, the platform facilities real-time integration of multiple simulation software, each modelling different aspects of a transit system (rail, surface transit and stations). Transit users, as agents are transferred between the various software as they travel through the network, with their overall route determined separately by a path choice decision making component. Outputs of the system include detailed logs and metrics produced by individual software (e.g. station LOS, rail performance metrics), and also full trip logs of agents and vehicle loads. (Srikukenthiran and Shalaby, 2017)

As this study is focused on bus network service tactical planning optimization, simulation in Nexus was limited to surface transit. The surface transit
simulator in Nexus was based off of its initial implementation, described in Srikukenthiran and Shalaby (2017). The network is constructed dynamically using information provided in the Google Transit Feed Specification (GTFS), which amongst others, describes stops, routes, schedules, and geographic paths taken by vehicles. Vehicles make a set of trips, blocked also based on information provided in GTFS. While for the initial model vehicle speeds were set to maintain vehicles being on schedule, this feature was adjusted in this paper, incorporating a random-forest speed prediction model utilizing historical automatic vehicle location (AVL) data available publicly; details on data preparation and speed prediction can be found in Wen et al (2018). Dwell times remain set based on a set formula as prescribed by Vuchic (2005) with an assumed boarding and alighting time per passenger of 3.0 and 1.5 sec, respectively. Transit user agents traverse this surface network as they make their way from origin to destination using an quickest-path (based on published schedules with a penalty for transfers) path-finding method.

As an agent-based microsimulation software, Nexus produces highly detailed output of all movements of both surface vehicles and transit user agents. Logs are kept of vehicle interactions at all bus stops; information recorded include the vehicle and trip number, arrival and departure times, incoming vehicle load, and the total # of agents boarding, alighting and unable to board due to a capacity limit. Each agent also keeps a record of their individual trips, with information including departure and arrival times, the boarding and alighting stop of each segment (both in-vehicle and transferring), vehicle trip taken, and any walking, waiting and in-vehicle duration.

4 Case study

To demonstrate the application of the limited-stop optimization method, an analysis was carried out based on a case study of the Hamilton Street Railway (HSR) transit network. The HSR serves over 80 thousand passengers daily. One of the major bus routes of the network, Route 1-King, was selected as shown in Figure 2. The route has two branches with over 12 thousand passengers daily ridership. The first branch which is the main one, serves between The GO Centre Platform and Eastgate Square. The second branch runs between University Plaza and Hamilton GO Centre. For this study the first branch is used. The route consists of 45 stops, and runs in a west-east direction in lower Hamilton from the McMaster Medical Centre in the west to the Eastgate Square in the east. The length of route is 12 km in each direction and the planned headway is 10 minutes during the morning-peak.

While a single route and direction was the focus of the study, a weekday morning peak-period (7:00-9:00AM) Nexus simulation model of the Greater Toronto and Hamilton Area (GTHA) region was utilized. Being a proof-of-concept model, the 2016 GTFS data were used to construct the regional transit system, while transit demand was provided by the 2011 Transportation Tomorrow Survey. The Transportation Tomorrow Survey is a regional travel
survey conducted every 5 years in the GTHA, which in part asks for a one-day travel diary, including detailed route information for those using transit. The data retrieval system for the TTS allows for extraction of O-D matrices in 10 min departure time slices; these matrices were used to generate an agent population for the region. Additional detail on the method used can be found in Srikukenthiran and Shalaby (2017). This larger regional model was used in place of a more focused single-route model given the prior construction/and availability of the regional model, as well as transit demand O-D being available only at the regional level, to avoid requiring assumptions to be made for travel to/from Hamilton. Figure 3 presents passenger demand profile for Route 1-King, at each stop in Eastbound direction.
5 Results and analysis

Modifications of the simulation model for consistency with the initial regional model are made for this study, with the implementation of the aforementioned random forest speed prediction model for more realistic movements of buses in the case study area. The optimization problem is coded in Python, and "Bonmin"-an open source optimization solver- is used to solve the problem. The computations are carried out on a Mac OS X machine with a 2.4 GHz Intel Core i5 processor and 8 GB of RAM. Table 1 summarizes the total objective function value and the computational times for the case study. The optimal solution is obtained after applying the branch & bound method within an optimal gap of solution (i.e., less than 5% gap).

As a baseline, the in and out-of-vehicle wait times contribute the same marginal time; however, different relative wait ratios (in-vehicle over out-vehicle times) are tested in the sensitivity analysis. The average walking speed of a pedestrian is 5km/hr and the average bus speed is 25km/hr according to the study data. The service schedule headway is fixed at 10 minutes over the time period. The bus capacity is 60 pass/veh, with 40 seated and 20 standing.

The detailed results obtained by the developed methodology are shown in Table 2. Each trip shows the optimum service pattern. In particular, it can be seen which stops are the most appropriate candidate for applying the LS strategy. The patterns illustrate that Stops, 23, 24, and 25 which are located in the middle of the route, are the most non-served stops for the majority of trips. The benefit of using this strategy yields saving times for each trip as indicated in Table 2. As can be seen, the results also show a somewhat superior outcome for trip #2 and #5 than for other trips, with a 20% and
Stop pattern recognition: The results of Table 2 demonstrate that a group of trips may follow a similar pattern. As implementing trip-by-trip skip stop operation would not be practically feasible, finding a small number of LS branches is necessary. Consequently, further analysis using a pattern recognition approach for the binary result data set is carried out. We select a binary pattern, that is [0 1] (meaning a served stop followed by a non-served stop).

The frequency of occurrence of the defined binary pattern for each trip is depicted in Figure 4. In the figure, those stops that follow the defined pattern are selected, meaning that the x-axis shows stops where the 0 of the pattern occurs, implying that the indicated stop will not be served (or is equal to 1 in the pattern). This will lead to reducing the search space needed to find the best combination of stops and the number of branches, by removing those stops that are not candidates for a LS strategy. As the figure illustrates there are two different trends of using stops. These trends are recognizable from the peak points. Peaks with value of three or more are considered in this study since the most frequent situation happens in the case of three or more number of trips. The first pattern is mostly shared among odd numbered trips, while the second pattern is mostly shared among even numbered trips. We name these two patterns as "Branch A" and "Branch B". Now we should assign the suitable stops to each branch utilizing the information of this figure. As an example, let’s assign the most frequent stops (22, and 24) to Branch A and the
Table 3: Summary of feasible branches

<table>
<thead>
<tr>
<th>Branches</th>
<th>LS pattern</th>
<th>Average time saving (passenger-sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>&quot;0000101010001000101010100000010000000000000000000000000000000000000000&quot;</td>
<td>1958.91</td>
</tr>
<tr>
<td>B</td>
<td>&quot;010000000000100000000000000000000000000000000000000000000000000000000000&quot;</td>
<td>13725.65</td>
</tr>
</tbody>
</table>

second frequent stop (23) to Branch B, respectively. This indicates that Branch A serves stops 22 and 24 while skipping stops 23 and 25. Similarly, Branch B serves stop 23 and skips stop 24. We then continue the same procedure for the other peaks (i.e., frequency \( \geq 3 \)) with a careful attention that stop assignment is due to the highest number of odd or even trip numbers that are sharing the stop where the pattern occurs. Consequently, all stop pattern assigned to both Branch A and B is obtained as follows, shown in Table 3. Each trip (of all 9 trips in our case study) then follows patterns in Branch A and B sequentially. *Sensitivity analysis:* A sensitivity analysis of the objective function (OF) value with weightings of passenger wait time and speed ratio is shown in Figure 5. The marginal burden or disutility of out-of-vehicle waiting time is perceived to be more burdensome than in-vehicle travel time (Reed, 1995). Thus, the wait time factor computes as in-vehicle over out-of-vehicle times (\( \delta_2/\delta_1 \)). A wait time factor less than 1 means the passengers who are not through passengers are likely to experience higher disutility of wait time with less saving time for the system.

The other factor when considering the application of a limited-stop strategy is the ratio of the average speed of a bus and the average walking speed of pedestrian (\( f \)), which takes the walking penalty time into consideration. An adjustment of this ratio means higher or lower passenger travel time than the base data. The bus speed variation also leads to vehicle running time variation. That is, a higher ratio means higher bus speed and less walking time penalty. The results of Figure 5 reveal a somewhat superior saving time for a lower speed ratio with a higher wait ratio. In particular, the higher saving time in the objective function is attained in lower walking time penalty and greater value of in-vehicle wait time.

6 Conclusions

This paper presented a new mathematical programming model which aims at finding the best candidate route stops for a limited-stop strategy. Within this study, we analyzed the benefits of implementing such a strategy for the purpose of minimizing the total passenger travel time. The adopted approach consists of two steps: optimization and simulation. An agent-based simulation framework is built to represent a real-life example and to generate random input data for the proposed optimization model. Based on the formulations for optimization an efficient algorithm for finding the best candidate is developed.
The model was applied in a case study of one route in Hamilton (Ontario), and a detailed discussion regarding optimum service stop pattern is provided. In order to furnish a practical solution for selecting the best feasible service pattern, a new pattern recognition method is developed. A sensitivity analysis of the objective function value with weighting passenger wait time and speed ratio is conducted. The computational results demonstrate the effectiveness of saving time for the greater value of in-vehicle wait time.

Ongoing research related to this study includes validating some of the assumptions made, developing a family of stop and service pattern strategies, and utilizing more accurate simulation models to measure the transit systems KPIs.

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