Minimising transfer penalty in public transit networks with Constraint Programming

Rejitha Nath Ravindra · Mark Wallace · Daniel Harabor · Chris Loader · Ilankaikone Senthooran

Abstract

In public transport (PT) operations planning, transfers play a vital role in enabling efficient travel between origins and destination that are not connected by a direct service. Convenient transfers are the consequence of timetable coordination that minimises the passenger transfer waiting time between different PT services. However, a realistic timetable coordination problem is extremely complex as it involves mixed and conflicting objectives and constraints. In this paper, we present a constraint based optimisation model and apply it to the timetable coordination problem for a subset of PT network at the City of Wyndham in south western Melbourne. The objective of our study is to minimise excessive waiting time when transferring between bus and train services. The requirements are drawn from planners at Transport for Victoria (TfV) in Melbourne, reflecting the practical concerns of transport professionals. The model developed entirely in MiniZinc, is designed to accommodate a wide range of planning and operational scenarios that occur in reality. Moreover, it captures the coordination problem holistically, rather than optimising transfers one after another. As a theoretical contribution we provide results from a set of such scenarios that addresses the complexity of PT trip interactions and provides coordinated and cost-effective schedules. From a practical perspective, by offering robust solutions to the real concerns of schedulers our study seeks to bridge the gap between scheduling in practice and in principle.

Keywords Public transport · Constraint Programming · Timetable Coordination · MiniZinc

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1 Introduction

Transfers between multimodal public transport (PT) services play a vital role in connecting the broadest number of origins and destinations that are not connected by direct services (Mees, 2000). Many studies have recognized facilitating efficient multimodal transfers as a key factor in improving the service performance of a transit system (Currie & Loader, 2010; Nielsen, 2005). However, when passengers are required to make transfers between different services to complete their journey, transfer waiting time is incurred. This waiting time imposed on transferring passengers is a notable negative side of transfers and it could be a major hindrance to both current and potential transit users. Timetable coordination is hence an essential planning and operational strategy that minimises the passenger transfer waiting time between two or more services, at an acceptable transfer penalty.

Transfer penalty in PT systems measures the disutility of a transfer during a trip in comparison to a non-transfer alternative (Guo, 2003). The common inference from many studies is that transfer penalty is the reluctance of passengers in making a transfer and a major impediment to PT use (Currie & Loader, 2010; Douglas & Jones, 2013; Guo & Wilson, 2004). The major focus behind identifying transfer penalty as a coordination objective and minimizing it is to promote ridership and improve vehicle coordination (Wong, 2000). In this study, we define transfer penalty as the disutility of passengers having to experience excessive waiting times at a transfer station and aim to minimise it across the network. We study the case of bus to train transfers as it is identified as the most efficient way to travel for many journeys in large cities.

The major challenge with solving the timetable coordination problem is the mixed and conflicting objectives and constraints associated with it in the real world (Ceder, Golany, & Tal, 2001). The problem of minimising transfer penalty is combinatorial, with a search space of candidate solutions that grows exponentially with the size of the problem (Klemt & Stemme, 1988). While computerized systems can help trade off network coordination of timetables to a certain extent, it is highly complex to achieve satisfactory solutions in a multiple route scenario. In spite such complexity with the problem, currently, coordination problems are solved by a combination of existing automated methods and manual efforts. Herein lies the need to develop a holistic and smarter algorithm that incorporates realistic constraints to solve the timetable coordination problem efficiently.

Drawing from requirements prioritized by our industry partners in Transport for Victoria (TfV) in Melbourne, Australia, we have created a constraint based optimization model aimed at minimising transfer penalty for transferring passengers across the PT network. Developed entirely in MiniZinc, an expressive constraint modelling language (Nethercote et al., 2007), the model precisely captures mixed and conflicting objectives and constraints. The underlying state of the art lazy-clause generation (LCG) solver (Feydy & Stuckey, 2009; Senthooran, Wallace, & De Koninck, 2015) is capable of solving the problem holistically. Moreover, the model is flexible to accommodate a wide range of “what-if” operational scenarios that occur in reality (Harabor & Stuckey, 2016). From a practical perspective, through a versatile and holistic algorithm to optimize timetables, planning and operation agencies can realise accurate, realistic and cost-effective solutions. Ultimately, this study seeks to bridge the gap between scheduling in practice and in principle.

The rest of the paper is organised as follows. In Section 2, we briefly describe the essential coordination requirements that arise in a Melbourne context, with some highlights on a PT subnetwork used for our study. Section 3 describes our timetable coordination problem, listing the real-world constraints. The problem is then formalised in Section 4 where we explain solution generation with a Constraint Programming based optimisation model using real world coordination constraints. In Section 5 we present Numerical Experiments and Results after applying the model for the given PT subnetwork. Following the inferences from our study, in Section 6, we briefly contrast related works that study the different approaches and challenges involved with achieving improved timetable coordination. The paper concludes with a discussion of the major findings and some directions on future research in Section 7.

2 The Wyndham Subnetwork

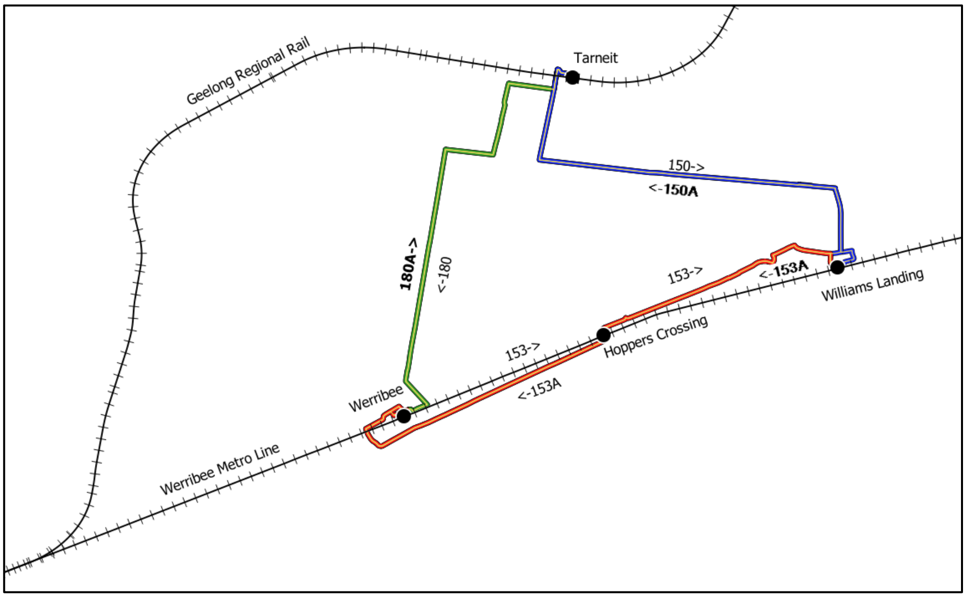

Fig 1. Wyndham Sub-network

This work results from a collaboration between academic researchers and transport planners at Transport for Victoria (TfV) in Melbourne. The team explores timetable coordination at a subset of a real-world PT network: City of Wyndham in the outer south-western Melbourne, located between Melbourne Central Business District (CBD) and regional city of Geelong. Its PT system includes 22 bus routes and 5 train stations in two rail lines – the Werribee Metro Line and the Regional Rail Link of Geelong. The subnetwork in our scope comprises 3 bidirectional bus routes and 4 train stations that enable multimodal transfers (Figure 1). Observed unevenness between bus and train frequencies at different time of the day, location and route direction makes timely coordination a highly challenging task in Wyndham.

Most bus routes in Wyndham have an existing average bus frequency of 20-40 minutes during the day on weekdays. Bus frequencies are targeted to meet passenger demand for many different travel purposes and train headways are irregular in peak time. Such variation makes achieving timely bus-train coordination at a reasonable cost highly challenging.

The most important and realistic coordination constraints with regard to achieving improved timetable coordination at Wyndham network are thus identified as follows:

(1) Transfer waiting time: We allow the transfer waiting times to comprise a minimum walk time to the transfer station plus some contingency time that ideally varies by location, time of day, type of day and direction of travel. This range must not be too short to risk missed connections, and not too long to waste time.

(2) Variable service headways: We allow some tolerance in bus headways from target headways to ensure meeting with intersecting trains. A minimum and maximum headway range for each route is specified, that also varies between peak and inter-peak periods. Buses also serve non-transferring passengers and the imposed maximum headway on each route meets these needs, dependant on the time of the day.

(3) Operator imposed constraints: In order to improve coordination and use bus fleet size in a multiple route scenario efficiently, we consider operator imposed constraints such as the

layover time and deadheads (unproductive time involved with a bus trip) (Fleurent, Lessard, & Séguin, 2004).

(1) Interlining: Buses are normally not constrained to only operate on one route. We allow buses to move through any route that provides a better opportunity to achieve coordination and satisfy the service requirements.

Through numerical experiments in Section 5, we provide an instantiation of solving the timetable coordination problem using the above listed real-world constraints.

3 Problem Description

Consider a multimodal bus and train transport network as shown in Figure 2(a). Routes are defined between transfer stations, with a separate route in each travel direction. At a coordinating transfer station, passengers on buses alight at the station, walk to the platform and transfer to the next available train. Hence, the minimum required time to make a successful transfer from a bus arrival to the next available train is the walk time plus minimum acceptable contingency time. We set the end of the schedule period so that there is always such a train. The major challenge is to achieve improved timetable coordination with multiple, conflicting objectives and constraints that reflect a number of real world scenarios. Based on industry discussions with TfV, we model the coordination problem with a few such requirements as elaborated in Section 3.1.

3.1 Coordination Requirements

3.1.1 Bus Task

Each bus route in the network has a specified running time between transfer stations. On completion of a trip on a given route, the bus then has a “layover” time before the start of its next route. This acts as a buffer to account for driver breaks, late bus arrivals due to various factors such as congestion, road conditions, delayed departure etc. Henceforth we will include the layover time in the runtime for a trip. In the simplest case the next route starts at the location where the previous route ended. As an exception, however, the next route might start from a different location. If an improved coordination efficiency can be gained by allocating a different route to the same bus, this can require some inter-route travel time called the deadhead time (running without passengers).

As data for our model we add deadhead time between each pair of routes as a conscious decision to support the optimisation model. Thus a single bus could make trips on a variety of different routes over the schedule horizon, and the sequence of routes made by a bus is selected by the optimiser. There is naturally a constraint for a bus that the time required between completing one route and starting the next is greater than the sum of the layover time plus the deadhead time. If this time is indeed more than the layover plus deadhead time, then this difference is wasted time for both the bus and its driver, and it is accordingly minimised by the optimiser. Nevertheless due to constraints on bus fleet size, bus route headways and bus train coordination, the overall optimum solution might include wasted time for some buses and drivers during the schedule.

For example, consider a simple network segment as shown in Figure 2(b). Let’s assume a bus completes its first trip A at station 2 and starts its next trip B at station 4. The bus finishes route A, has a certain layover at station 2 and deadheads to station 4 to start its next route, B. Let’s denote the time between the end of the trip A and the end of the trip B as “*task time*”. Hence, as a data input for our model, the task time for this network segment is calculated as:

$$Task\ Time_{A,B} = dh_{A,B} + rt_B \quad (1)$$

Where, $Task\ Time_{A,B}$ is the time between the end of trip A to the end of trip B

$dh_{A,B}$ is the bus deadhead time between trips A and B

rt_B is the runtime for trip B

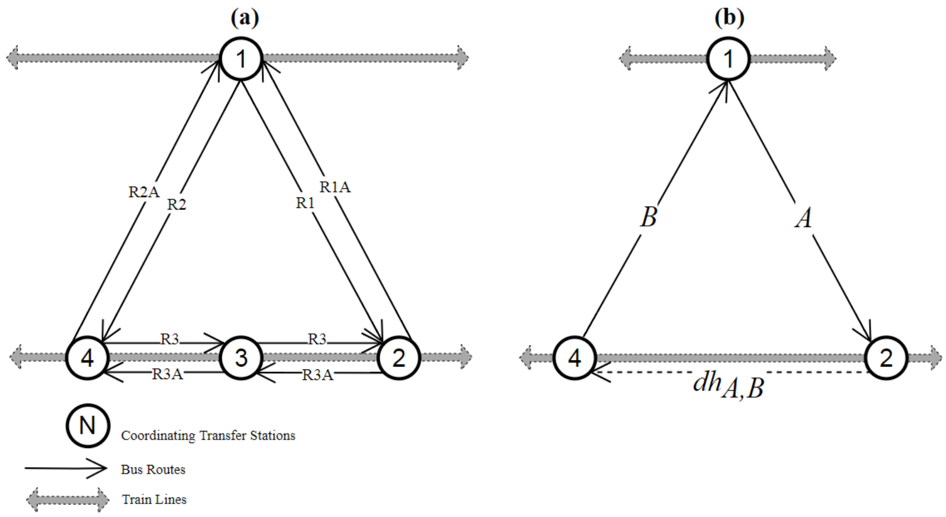

Fig 2. (a) Configuration of a multimodal PT network (b) Task time for a single network segment

3.1.2 Bus headway

In transit systems, headway refers to the separation time between two consecutive trips on a single route (Ceder, 2007). In practice, shorter passenger wait times can be achieved by shortening the bus headways at the cost of dispatching additional number of buses. Alternatively, headways could be lengthened with fewer buses but at the cost of incurring longer waiting time at transfer locations. Planners are usually left with intuitive judgement in deciding the frequency settings that simultaneously minimise transfer wait times with the least number of buses used (Currie, 2009).

As a case study, we consider some flexibility in bus headways since it is disadvantageous to demand a perfectly consistent headway throughout the schedule horizon. Sometimes small variations in bus headways can enable vehicle scheduling efficiencies that would not be possible with strict even headways. Also, in some instances train headways are uneven, particularly in peak periods, and this may require flexibility in bus headways to ensure good connections across multiple trips. The acceptable minimum and maximum bus headway values are determined at a certain tolerance value from operator defined target headways, and different acceptable ranges may be trialled as part of "what if" scenario analysis. It must be noted hence that flexible headways can result in "bus bunching" and we avoid this by introducing time constraints for the first and last bus trips.

3.1.3 Time dependent transfer passenger volume

The transfer volume between PT services varies by time of day. Passenger volumes can be incorporated into the optimisation objective function to higher levels of coordination at the busiest times of day. Moreover, with flexible headways, the transfer waiting time incurred by different groups of passengers will be different.

Figure 3 illustrates how we have modelled transfer volumes by time of the day. Different from the usual approach of considering an average or aggregate value for transfer volume for a given time of day, we calculate the proportion of transfer volume between two consecutive bus arrivals from a given hourly volume at each station.

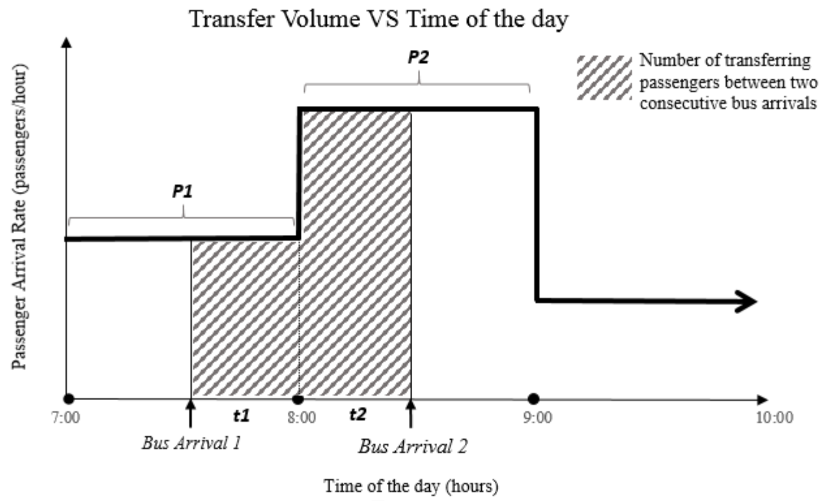

Fig 3. Varying transfer volume with time of the day

From the figure, the transfer volume between consecutive bus arrivals 1 and 2 is given as:

$$Transfer\ Volume,\ P = P_1 * t_1 + P_2 * t_2 \tag{2}$$

Where P_1 = Passenger Arrival rate at hour 1 (passengers/hour)
P_2 = Passenger Arrival rate at hour 2 (passengers/hour)
t_1 = Interval between bus arrival 1 and the next hour (hours)
t_2 = Interval between bus arrival 2 and the previous hour (hours)

4 Model Formulation

In this Section, we present a constraint based optimisation model to minimise the transfer penalty incurred by transferring passengers at all transfer stations in the network.

4.1 Model Assumptions

1. The coordination priority at each station is pre-determined by planners based on demand, bus route direction, time of day and type of day.
2. There is one priority transfer station per route direction.
3. Train departure times at transfer stations is an input.
4. The number of bus trips on each route in a given time period is fixed.
5. Runtimes and layover times are fixed for all routes.
6. Trains and buses have sufficient capacity to meet demand.

4.2 Sets and Parameters

Let's define the given network as G = {S, R} with S set of transfer stations and R set of single-direction bus routes.

Input Data

T	:	Set of all train departures in the network
R	:	Set of all bus route directions in the network
S	:	Set of all transfer stations in the network
PT_s	:	Set of physical trains departing from station s
B	:	Set of all physical buses in the network
TR	:	Set of all bus trips in the network
T_0	:	Start time of the schedule horizon

T_{max}	:	End time of the schedule horizon
$TD_{s,pt}$	:	Train departure time for each train pt from station s
r_s	:	Transfer station s at each route end
$walk_r$	:	Walk time for passengers from bus to train at a route end r, $r \in R$

Problem Data

bct	:	Bus fleet size
$btrips$	:	Maximum number of trips by any bus in T_{max}
$TT_{r1,r2}$	:	Task time between transfer station on route *r1* and transfer station on route *r2*; $r1, r2 \in R$
$Hmin_r$	:	Minimum bus headway on route r, $r \in R$
$Hmax_r$	:	Maximum bus headway on route r, $r \in R$

4.3 Decision Variables

We introduce the following variables that produce a bus schedule that minimises the total passenger transfer waiting times. The solver chooses the bus arrival time at the end of each trip and the vehicle which operates it.

$\forall \quad r \in R, t \in TR,$ $\quad BN_{r,t}$ = Physical buses assigned to the t^{th} bus trip on r^{th} route
$\quad BA_{r,t}$ = Arrival time of the t^{th} bus trip on r^{th} route at the transfer station

Dependent Variables:

$\forall \quad r \in R, t \in TR,$ $\quad P_{r,t}$ = Number of transfer passengers from t^{th} bus trip on route r to the next connecting train at a transfer station

$\forall \quad s \in S, pt \in T,$ $\quad TD1_{s,pt}$ = Train departure from station s that follows immediately after a bus arrival at r^{th} route on its t^{th} trip plus minimum walk time.

$\forall \quad r \in R, t \in TR,$ $\quad Penalty_{r,t}$ = Transfer penalty expressed as the excessive passenger waiting time summed over all routes *r* and trips *t* in the network

4.4 Constraints

I. **Task time Constraint**: For a single bus traversing on the same or different routes $r1, r2$ the arrival time on its next trip is after the previous trip by at least the task time between the two successive trips.
   i. If bus trip t1 on r1 happens before trip t2 on r2 i.e., $BA_{r1,t1} < BA_{r2,t2}$
      Then $\quad BA_{r1,t1} + TT_{r1,r2} \leq BA_{r2,t2}$
   ii. If bus trip t2 on r2 happens before trip t1 on r1 ie $BA_{r1,t1} > BA_{r2,t2}$
      Then $\quad BA_{r2,t2} + TT_{r2,r1} \leq BA_{r1,t1}$

$$\forall \; r1, r2 \in R,\, t1, t2 \in TR,\, BN_{r1,t1} = BN_{r2,t2} \qquad (1)$$

II. **Headway Constraints:** The headway between successive bus trips $t + 1$ and t on a single route r is constrained to fall between a minimum headway- $Hmin_r$ and a maximum headway- $Hmax_r$.

$$Hmin_r \leq (BA_{r,t+1} - BA_{r,t}) \leq Hmax_r \qquad \forall\, r \in R, t \in TR \qquad (2)$$

III. **Passenger Transfers:** Consider a schedule horizon spanning from T_0 to T_{max} divided into blocks of *h-1, h, h+1*... hours with corresponding hourly transfer passenger volume of P_{h-1}, P_h, P_{h+1} etc. The transfer volume $P_{r,t}$ between consecutive bus arrivals is calculated for the following cases:

i. If the consecutive bus arrivals fall in the same hour block, i.e.

a) $BA_{(r,t)} < h$ then $P_{r,t} \geq (BA_{(r,t)} - BA_{(r,t-1)}) * (P_h/60)$

b) $BA_{(r,t-1)} > h$ then $P_{r,t} \geq (BA_{(r,t)} - BA_{(r,t-1)}) * (P_{h+1}/60)$

$$\forall\, r \in R, t \in TR, h \in T_{max} \qquad (3)$$

ii. If the consecutive bus arrivals fall in different hour blocks, i.e. (Figure 4)

$BA_{(r,t)} > h \;\&\; BA_{(r,t-1)} < h$

then $P_{r,t} \geq \left((h - BA_{(r,t-1)}) * P_h/60\right) + (BA_{(r,t)} - h) * P_{h+1}/60\right)$

$$\forall\, r \in Rt \in TR, h \in T_{max} \qquad (4)$$

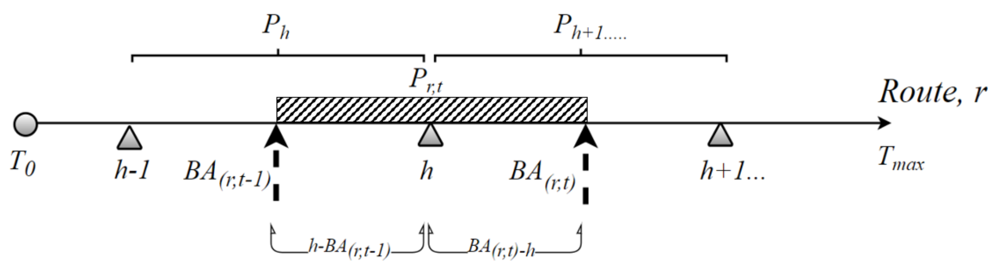

Fig 4. Passenger transfer as a function of bus arrivals

IV. **Wait time Constraint**. For all bus arrivals by trip t on route r, there exists a train departure from station s that immediately follows the bus arrival and minimum walk time.

$$TD1_{s,pt} = \min\{TD_{s,pt} : pt \in PT_s \;\&\; TD_{s,pt} \geq BA_{r,t} + walk_r\}$$
$$\forall\; TD_{s,pt} \neq Ntime, r \in R, t \in TR, s \in S \qquad (5)$$

From (5), for all non-null values of bus arrival and train departure, waiting time constraint can be expressed as:

$$TD1_{s,pt} \geq BA_{r,t} + walk_s \qquad (6)$$

V. **Penalty Constraint:** The transfer penalty is calculated as the sum of extra waiting time incurred by transferring passengers at all transfer stations in the network.

$$Penalty_{r,t} = (TD1_{s,pt} - BA_{r,t} - walk_s) * P_{r,t} \qquad \forall\, r \in R, t \in TR, s \in S, pt \in T \qquad (7)$$

VI. To ensure that the entire schedule horizon is covered by trips, we introduce first bus and last bus constraints as follows:

i. **First bus constraint:** The first bus trip on route r must arrive at a time between the beginning of the given schedule horizon T_0 and a certain maximum headway H_{max} since T_0.

$$T_0 \leq BA_{r,t} < T_0 + H_{max,r} \qquad \forall\, r \in R, t = 1 \qquad (8)$$

ii. **Last bus constraint:** The last trip of a bus on route r must depart from the station at a time between the maximum headway H_{max} and T_{max}.

$$T_{max} - H_{max,r} \leq BA_{r,t} < T_{max} \qquad \forall\, r \in R, t = btrips \qquad (9)$$

Objective: Based on the above constraints, the objective for minimising the total transfer penalty in a transit network can be expressed as follows:

$$Min\ \mathrm{Z} = \sum_{r,t} Penalty_{r,t}$$

5 Numerical Experiments and Results

In this section, we provide an instantiation of minimising transfer penalty using the constraints listed in Section 4.4 on a subset of a real-world PT network. As briefly introduced in section 2, we chose a triangular subnetwork from the City of Wyndham comprising of 3 bi-directional interlining routes coordinating with 1 transfer station on the Geelong line (V-Line, Regional Rail) and 3 transfer stations on the Werribee line (Metro). To implement the proposed model, we use input data as given in Table 1.

Table 1. Data setup and source

Data Input	***Description***	***Source***
Bus trips	Number of bus trips on each route	TfV timetables
Runtime	Runtime on each route	TfV timetables
Deadhead time	Deadhead time between each pair of routes	Google Maps
Target headway	Planner defined target headways for each route at different time periods of the day	TfV service specifications
Walk time	Walk time between bus stop and departure platform at each station	Google Maps
Layover time	10% of runtime as recovery time at the end of each trip	TfV timetables
Train timetable	Train departure time from each station	TfV timetables
Transfer passenger volume	Hourly transfer passenger volume at each transfer station	PT smartcard (Myki) data

Some model and network specific features sourced from industry specifications are as follows. An overview of these are also provided in Appendix A. The existing train schedule for Wyndham network that serves as an input to our model is shown in Appendix B.

- We consider a schedule horizon from 7:00 AM to 9:00 AM (morning peak) for weekday city-bound trips from Wyndham suburbs.
- We schedule a certain fleet of buses over 6 route directions to enable interlining between multiple routes.
- For peak hour trips, we consider that buses coordinating with city bound trains have primary priority.
- We do not consider train to bus connections due to lower connection priority in morning peak.
- For AM peak, note that routes 180A, 150, 153 and 180 have primary coordination priority with city bound trains while routes 150A and 153A have secondary priority and are only used to satisfy the headway constraints for all interlining routes.

5.1 Numerical Experiments

For each transfer station, we consider a minimum transfer time to allow for all transferring passengers irrespective of their physical capabilities to reach the departure train platform. Any transfer time beyond the minimum is considered excessive, and the sum of such times is the transfer penalty. Within this context, we consider the following objectives in our experiments:

- Objective 1: minimisation of transfer penalty exceeding the walk time
- Objective 2: minimisation of transfer penalty exceeding the walk time plus industry specified slack time of 5 mins

We add a minimum layover as recover time at the end of each trip. As an industry requirement, we define minimum layovers as 10% of route runtime. We then contrast our penalty results for the following scenarios:

- Scenario 1: with layover time (buses arriving late) and
- Scenario 2: without layover time (buses arriving on time) scenario.

Implementation: A Constraint Programming based algorithm was used in the implementation of our model. The model is written entirely in MiniZinc and tested using the Chuffed solver (Chuffed, 2016).

We run our test scenarios with multiple bus fleet sizes for a maximum of 30 minutes. To run the case studies, our model and parameters remained the same as in section 4 with only simple modifications to the required data. Such data-driven modifications make it easier to explore a variety of scenarios in our model. We investigate our objectives with the given cases on fixed and variable headways as given in Table 2.

Table 2: Case study- Minimum and maximum headway for each route

Bus Route	*Case 1: Fixed/Target Headway (mins)*	*Case 2: Variable Headway (mins)*	
	Fixed	*Min*	*Max*
180A	20	16	24
150	20	16	24
153	40	32	48
180	20	16	24
150A	20	16	24
153A	40	32	48

Case I: Transfer penalty with fixed headway

It is a common practice for transit agencies to base timetables on even and fixed headways - and this would significantly reduce flexibility in the coordination task, as the only choice would be the timing of the first bus trip. However it makes it very difficult to achieve good coordination if train headways are not perfectly even.

We model this case by modifying only the input headway data to take fixed target values as given in Table 2. Results from this case are given in Table 3(a). Figures 5(a) and (b) show the results graphically. The solver was able to find and prove optimality for all feasible solutions with execution times spanning from a few seconds to 5m 43s. Note that “U” represents instances where there is no feasible solution (due to too few buses).

It can be observed that with fixed headway, for a network of the given size, bus fleets of 6 or fewer buses would not suffice in meeting the specified requirements with or without slack time. Looking at objectives 1 and 2 individually, there is only slight difference in the transfer penalties with and without layover time. But contrasting objective 1 with 2, although the fleet size requirement remains the same, it can be seen that the penalties for corresponding number of buses significantly reduce when a slack time of 5 mins is allowed beyond walk time. When looking at the trade-off between minimised penalty for passengers and bus fleet size requirements, the best solution for this case study is obtained for objective 2, without any layover (2430 pass-mins at 8 buses).

Table. 3 (a) Case I: Optimised Total Transfer Penalty with Fixed Headway

Bus fleet size	Fixed headway							
	Objective 1: Penalty > Walk				Objective 2: Penalty > Walk + 5			
	Scenario 1		Scenario 2		Scenario 1		Scenario 2	
	*Penalty	**Time (s)	Penalty	Time (s)	Penalty	Time (s)	Penalty	Time (s)
5	U	1	U	1	U	1	U	1
6	U	1	U	1	U	1	U	1
7	11856	343	10594	164	2915	102	2566	43
8	9223	210	9131	88	2435	68	2430	8
9	9131	92	9131	28	2430	18	2430	7
10	9131	67	9131	40	2430	15	2430	5
11	9131	38	9131	43	2430	6	2430	5
12+	9131	16	9131	32	2430	3	2430	3

*** Penalty is expressed in passenger-minutes*

**Time (s) is the solver execution time taken to find and prove optimality*

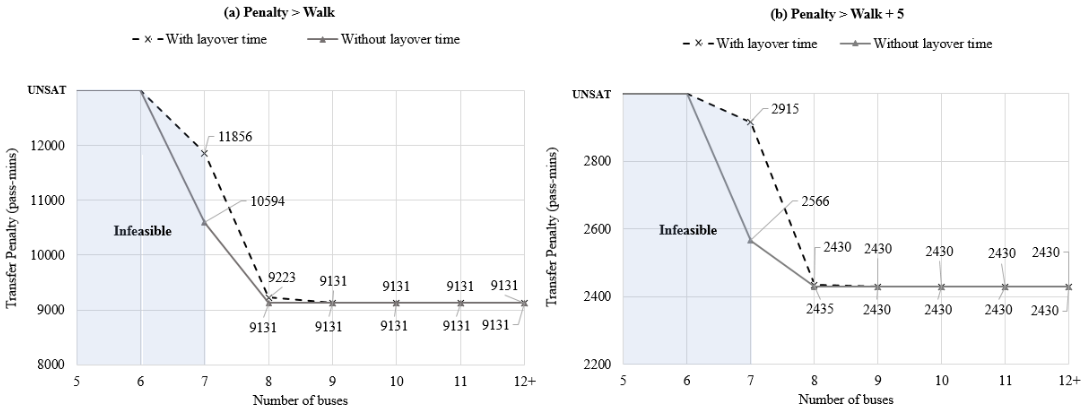

Fig 5. Optimised total transfer penalty vs bus fleet size for fixed headway, (a) Objective 1: Penalty > Walk (b) Objective 2: Penalty > Walk + 5

Case II: Variable Headway

To allow a reasonable degree of flexibility in bus headways, we permit the headways to span between a minimum and maximum range, defined at 20% tolerance from the target headway for each route (Table 2). These bus headway are distributed between the earliest train at 7am and the latest train before 9am. Results from this case are given in Table 3(b). Figures 6(a) and (b) show the transfer penalties obtained graphically. The solver proved optimality for all feasible solutions with execution times spanning from a few seconds to 26 minutes.

Interestingly, the results show that if we were to consider no layover time to either objectives, the entire network can be run with one less bus (#6) than with layover (#7) but, at the cost of incurring higher penalties to passengers (18533 pass-mins and 3280 pass-mins for objectives 1 and 2 respectively). This could be a significant saving in operator cost but at a higher transfer disutility to passengers. Contrasting the objectives, there is significant reduction in penalties when a slack time is added beyond walk (objective 2). Moreover, while having some layover gives us better coordinated timetables (lesser penalty) at the cost of an extra bus, not considering it could require fewer number of buses at the cost of higher penalty. Also notice that in objective 2 without layover, the penalties are not much different between using 7 and 8 buses (400 pass-mins and 312 pass-mins respectively). Any number of buses more than 8 incurs same penalty and is a waste of resource. In such a case, as a cost efficient option, a planner may decide to choose 7 buses than 8. We present the optimised bus schedules for these two options in Appendices C and D.

Table: 3 (b) Case II: Optimised Total Transfer Penalty with Variable Headway

Bus fleet size	Variable headway							
	Objective 1: Penalty > Walk				Objective 2: Penalty > Walk + 5			
	Scenario 1		Scenario 2		Scenario 1		Scenario 2	
	Penalty	Time (s)	Penalty	Time (s)	Penalty	Time (s)	Penalty	Time (s)
5	U	1	U	1	U	1	U	1
6	U	1	18533	602	U	1	3280	400
7	8911	1560	6831	519	678	206	400	34
8	6433	1560	5725	384	400	32	312	20
9	5757	1200	5378	108	312	4	312	6
10	5378	122	5378	78	312	6	312	12
11	5378	64	5378	76	312	3	312	7
12+	5378	82	5378	78	312	7	312	9

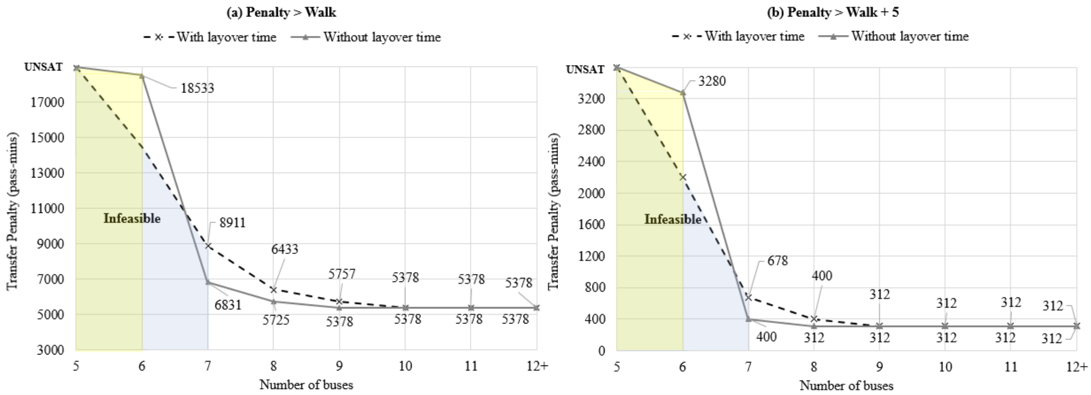

Fig 6. Optimised total transfer penalty vs bus fleet size for variable headway (a) Objective 1: Penalty > Walk (b) Objective 2: Penalty > Walk + 5

Comparing case studies I and II, we observe that relaxing some coordination requirements can have significant impact on the total transfer penalties incurred by passengers and the bus fleet required to serve the entire network. Allowing some flexibilities to the requirements like target headway, layover and transfer waiting times resulted in lesser total penalties hence lesser passenger minutes wasted. Moreover, it is notable that our model was able to reduce penalties and hence improve timetable coordination at the network level, but also do so with least number of buses needed to meet the service requirements. Such a tool is essential in delivering better understanding of the trade-off between good timetable coordination and cost efficiency.

6 Related studies

Following the inferences from our study, in this section we briefly contrast related works and the various approaches in minimising transfer penalty in PT systems. Transfer penalty in PT systems measures the disutility of a transfer during a trip in comparison to a non-transfer alternative (Guo, 2003). The disutility generated by need to transfer between PT modes is measured by two different factors: waiting time at the departure location and reliability of the connection. Keudel (1988) identified the major objectives to planning transfer conditions as:

- minimising waiting times for transfer passengers across the network and/or
- minimising waiting times exceeding a certain comfort value (for example, 5 minutes)

At each transfer location, schedules for two different transport services must be linked as closely as possible. However each individual schedule is subject to its own customer service requirements and cost constraints. While devising the constraints for the problem is less difficult, achieving a trade-off between these constraints is hard. For example, as Currie and Bromley (2005) noticed, optimising passenger wait times often conflict with optimising fleet size and vehicle allocation since the former requires that vehicles spend time waiting whilst the latter requires faster speeds and shorter waiting to save operator costs. Moreover, a network wide coordination problem includes multiple transfer points, and coordination at one point carries forward to coordination at others due to these constraints. Klemt and Stemme (1988) identified that the resulting coordination problem is combinatorial and soon reaches the limits of manual handling. Hence, a more effective and realistic solution can only be achieved with computer aided methods.

A major barrier to obtaining efficient and coordinated schedules using traditional mathematical approaches is the highly complex nature of the timetable coordination problem. The search for optimal solutions is difficult to solve using traditional computational techniques (Cevallos & Zhao, 2006; Guihaire & Hao, 2008), and can require days of running time for even a small transit network. The complexity intensifies when user and operator requirements conflict. While users require service reliability, ideal waiting times and smooth transfers, operators aim at maximising vehicle utilisation.

The majority of studies simplify the problem by fixing certain operating conditions and use a combination of automated methods and human guidance. Most automated methods found in literature use MIP (Mixed-Integer programming) and MINP (Mixed-Integer Nonlinear Programming). These methods often involve unrealistic simplifying assumptions (Poorjafari & Yue, 2013) that make the timetable brittle and reduce overall PT reliability. Examples of such assumptions include:

- Constant headway along a given service line
- Unproductive service times which impact PT cost, such as layover time and deadheading, are discarded
- Bus to train transfer demand at a station is considered an average or aggregate value for any given time of the day.

In Constraint Programming (CP), relations between decision variables are stated as a set of constraints. While early works on transit network design using CP exist (Guihaire & Hao, 2008) they did not address PT timetable coordination. CP is a powerful tool to solve scheduling problems since it is able to precisely capture real world requirements which are difficult to express in MIP. Moreover it allows high-level, faster modelling of combinatorial feasibility and optimization problems. CP search, which uses a combination of variable and value-selection heuristics to guide the exploration of search space, can adapt to almost any form of constraint and objective (Belov, Stuckey, Tack, & Wallace, 2016; Pesant, Quimper, & Zanarini, 2012).

Inferring from the above discussions, timetable coordination is an extremely challenging optimisation problem. In spite of the complexity, it is solved using techniques that often involve specialised cases with simplified models and fewer constraints in ideal operating conditions. The underlying assumptions to simplify the problem often result in models which cannot represent real world transit systems. It can hence be concluded that to obtain efficient and realistic solution, PT timetable coordination relies heavily on Operations Research (OR) methods. Effective models that address the inherent complexity associated with timetable coordination problem are an important tool for public transport planners. It is within this context that this research was undertaken to present a constraint based optimisation model that minimise excessive transfer waiting times incurred by passengers transferring from a bus service to a train service across the network.

7 Discussion and Conclusions

This study focussed at achieving efficient timetable coordination by minimising total transfer penalty incurred by passengers transferring between bus and train services across a given network. The inherent complexity associated with timetable coordination is due to wide-ranging real world constraints and conflicting objectives. The data-driven nature of our model written entirely in MiniZinc and its ability to accommodate a wide range of such “what-if” scenarios makes it possible to capture such constraints realistically. It is notable that our model is not only able to provide optimal solutions for timetable coordination but also do so with the minimum number of buses that deliver an overall efficiency in utilising the given fleet size to its full potential. This can contribute to significant savings in operator cost. In this study, we provide an instance of minimising transfer penalty for a subnetwork in Wyndham. However, the actual reduction in transfer penalty cannot be measured in comparison to the existing schedule as the existing Wyndham bus fleet covers a much larger network. Hence, this study can be looked at as a scenario based subset of a bigger problem. We plan to extend this study to consider Wyndham network in its entirety.

Hence, as a theoretical contribution, we present a model that is capable of analysing a range of scenarios that address the complexity of PT vehicle interactions. It provides cost-effective schedules that are well coordinated temporally. We could extend our model easily to further investigate a number of scenarios of practical interest, such as a 24 hour schedule horizon; time-of-day dependent bus headways, running times and coordination priority directions; bus driver shift, break requirements and consequent costs. From a practical perspective, our model is designed to aid faster decision making to realise accurate, realistic and cost-effective solutions. Ultimately, this study seeks to bridge the gap between scheduling in practice and in principle.

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APPENDIX

Appendix A. Overview of Wyndham network*

Bus Route	Coordinating station (Number)	Route Direction	Coordination Priority	Number of bus trips on each route
180A	Tarneit (1)	Out	1°	6
150	Williams Landing (2)	In	1°	5
153	Hoppers Crossing (3)	In	1°	3
180	Werribee (4)	In	1°	5
150A	Tarneit (1)	Out	2°	6
153A	Hoppers Crossing (3)	Out	2°	2

* *Time of day: AM Peak,*

Appendix B: Train schedule for Wyndham network (as of January 2017)

Bus Route	Coordinating Station	Train Departure Times (h:mm) Weekday (Mon-Fri), 7:00 AM to 9:00 AM Direction: To Melbourne CBD											
180A	Tarneit	7:04	7:17	7:32	7:42	8:00	8:11	8:31	8:49	-	-	-	-
150	Williams Landing	7:02	7:11	7:17	7:26	7:36	7:46	7:56	8:08	8:18	8:30	8:50	8:51
153	Hoppers Crossing	7:07	7:13	7:22	7:32	7:42	7:52	8:04	8:14	8:26	8:36	8:47	-
180	Werribee	7:04	7:10	7:19	7:29	7:39	7:49	8:01	8:11	8:23	8:33	8:44	8:49

Appendix C: Optimised bus schedule with variable headway and 7 buses

Objective 2: Penalty > Walk + 5 Scenario 2: Without layover time #Bus= 7			
Bus Route	Bus Trips	Bus Arrival (h:mm)	Bus headway (h:mm)
180A	*1*	7:11	-
	2	7:31	0:20
	3	7:54	0:23
	4	8:10	0:16
	5	8:26	0:16
	6	8:43	0:17
150	*1*	7:15	-
	2	7:39	0:24
	3	8:02	0:23
	4	8:24	0:22
	5	8:44	0:20
153	*1*	7:00	-
	2	7:38	0:38
	3	8:25	0:47
180	*1*	7:03	-
	2	7:27	0:24
	3	7:49	0:22
	4	8:12	0:23
	5	8:36	0:24
150A	*1*	7:03	-
	2	7:25	0:22
	3	7:41	0:16
	4	8:05	0:24
	5	8:26	0:21
	6	8:48	0:22
153A	*1*	7:26	-
	2	8:12	0:46

Appendix D: Optimised bus schedule with variable headway and 8 buses

Objective 2: Penalty > Walk + 5 Scenario 2: Without layover time #Bus= 8			
Bus Route	Bus Trips	Bus Arrival (hh:mm)	Bus headway (hh:mm)
180A	*1*	7:14	-
	2	7:38	0:24
	3	7:54	0:16
	4	8:10	0:16
	5	8:30	0:20
	6	8:48	0:18
150	*1*	7:15	-
	2	7:39	0:24
	3	8:03	0:24
	4	8:27	0:24
	5	8:50	0:23
153	*1*	7:00	-
	2	7:37	0:37
	3	8:25	0:48
180	*1*	7:11	-
	2	7:32	0:21
	3	7:51	0:19
	4	8:15	0:24
	5	8:38	0:23
150A	*1*	7:15	-
	2	7:38	0:23
	3	7:54	0:16
	4	8:10	0:16
	5	8:30	0:20
	6	8:48	0:18
153A	*1*	7:26	-
	2	8:12	0:46

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