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Multi-Directional Transfer Time Optimization at a Single Transfer Node

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Abstract Most studies on transfer synchronization focus on modifications to the departure time of the first vehicle of each line from a terminal in order to minimize transferring waiting time or maximize the number of successful transfers with or without considering the associated demand at the network level. However, there is a critical need for a detailed investigation of the transfer process at an individual transfer node. This paper develops and compares three mixed integer programming formulations to assess the necessity of including transferring demand and vehicle capacity limit in a modelling framework of transfer synchronization. The models are tested through a numerical example. The results demonstrate that without proper consideration of the transferring demand and vehicle capacity unrealistic and unreliable optimal solutions may result.

Keywords: Public transit reliability · Multi-directional transfer · Single transfer node · Transfer demand · Transfer time optimization · Capacity Constraint

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1 Introduction

Due to the complexity of the planning process of a transit network, it is usually broken down into several smaller problems. In general, public transport operational planning consists of five basic steps (Haghani and Shafahi 2002; Haghani et al. 2003): transit network design, timetable design, vehicle scheduling, driver scheduling, and fleet maintenance scheduling. Each step provides information and constraints to the next step, although it is ideally preferred to consider all steps simultaneously. One of the objectives in transit network and route planning is to improve network connectivity through the provision of transfer nodes that connect efficiently the routes serving critical locations in terms of passenger load or/and zone importance. Well connected routes with reliable and synchronized transfers can offer commuters access to a diverse network with a variety of travel paths to accommodate their complex trip chains. Nevertheless, transfers represent a disutility for travellers due to the associated travel interruption and the time and effort required to walk to a platform of another route and wait for the next vehicle.

Transfer time optimization is one of the critical elements of bus timetable design from two primary viewpoints. The first is from the perspective of passengers. In most transit systems, many passengers have to undertake at least one transfer to complete their trip, during which they may experience long waiting times in the case of non-synchronized timetables (Petersen et al. 2012). Additionally, it has been shown that transit users consider transfers as one of the major factors that determine whether they use public transit (Ceder 2016). The second viewpoint is that of agencies, as developing synchronized timetables is important in order to not only manage efficiently their resources such as vehicles and drivers (Ceder 2016), but also to provide reliable service to passengers. Reliable service can lead to both retaining existing passengers and attracting new ones, thus increasing revenues.

Synchronization is one of the most complex problems of transit scheduling (Ceder et al. 2001) and it has been studied intuitively and extensively for decades. Most of the studies considered all transfer nodes in a network simultaneously. Therefore, due to the inherent complexity, researchers employed various heuristic methods such as genetic algorithms, which may not ensure globally optimal solutions. Recently, transferring demand received more attention in various modelling efforts. While some papers considered the average demand per line, others investigated the associated transferring demand of each vehicle.

On the other hand, vehicle capacity has been considered in very few studies. Liu and Ceder (2017) consider the capacity variable in the joint modelling of transit demand assignment and synchronized timetable generation. Also, Liu and Ceder (2016) defined a decision variable representing the type of the vehicle in their model aiming to minimize both the observed load discrepancy and passenger waiting time. However, both papers assumed passengers are able to board the first available vehicle, implying no additional delays due to lack of spare capacity. To the best knowledge of the authors, vehicle capacity has not been investigated as a constraint for the successful transfer process at the planning stage, although it was used in very few studies in transfer synchronization at the operational stage (Nesheli and Ceder 2015)

The main objective of this study is to develop a new approach to modelling the transfer optimization problem at a single transfer node for constructing synchronized timetables that minimize the total transferring passenger-waiting time for all lines and directions. Three models are formulated in Mixed Integer Programming (MIP) and examined to explore the necessity and benefit of considering vehicle capacity as one of the constraints. The remainder of this paper is structured as follows. First, a review of related studies is presented. Then, the methodology and modelling formulation are explained. Last, the modelling results followed by conclusions are presented.

2 Background

Numerous studies have explored transfer synchronization analysis and optimization through a wide spectrum of variables, constraints, and methods as well as various types of data and assumptions. Table 1 provides a brief list of studies.

Table 1 Summary of recent papers dealing with transfer synchronization

Author (year)	Objective function	Demand (level)	Modelling method	Solution method
Ceder et al. (2001)	Maximize the number of simultaneous transfers	No	MILP	Heuristic algorithm
Eranki (2004)	Maximize the number of successful transfers	No	MILP	Heuristic algorithm
Kwan and Chang (2008)	Minimize transfer delay and dissatisfaction, minimize the deviation from existing timetable	No	Multi-Objective	NSGA-II with tabu search and simulated annealing
Shafahi and Khani (2010)	Minimize transfer waiting time	Yes, (line)	MIP	CPLEX, GA
Ibarra-Rojas and Rios-Solis (2012)	Maximize the number of synchronization	No	IP	Heuristic algorithm

Ibarra-Rojas et al. (2014)	Maximize the number of synchronization and minimize operation costs	Yes, (vehicle)	Bi-Objective, MILP	ε- constraint method
Aksu and Yilmaz (2014)	Minimize waiting time and transfer penalty	Yes, (vehicle)	IP	GA
Ibarra-Rojas et al. (2015)	Maximize number of synchronization and minimize bus bunching	No	Multi-Objective, IP	Heuristic algorithm
Wu et al. (2015)	Minimize the maximal passenger waiting time	Yes, (vehicle)	IP	GA
Liu and Ceder (2016)	Minimize expected passenger wait time, minimize observed load discrepancy	Yes, (vehicle)	Multi-Objective IP	Heuristic algorithm
Wu et al. (2016)	Maximize the number of passengers benefitting from transfers, minimize the deviation from the existing timetable	Yes, (vehicle)	Multi-Objective	NSGA-II
Fouilhoux et al. (2016)	Maximize total number of weighted synchronization	Yes, (line)	MIP	CPLEX
Tian and Niu (2017)**	Maximize the total level of service: transfer utility	No	Non-Linear IP	Dynamic programing

Note: All the papers applied their models at the network level.
* MILP: Mixed Integer Linear Programming
* NSGA-II: Non-Dominated Sorting Genetic Algorithms
** The focus of the study is on a single transfer node and only two transfer directions.

In previous research efforts aimed at creating synchronized timetables, different objectives have been pursued and accordingly different modelling methods have been used. The two common types of objectives were maximizing the number of successful transfers or minimizing the total transfer waiting time for a specific period of time. In 2001, Ceder et al. developed a mixed integer programming (MIP) model to maximize the number of simultaneous bus arrivals. Eranki (2004) extended this study by defining a safe time window for bus arrivals for successful transfers instead of simultaneous arrivals. Later, Ibarra et al. (2012) presented an improved version of Eranki's model by adding flexibility to allow for a distribution of the departure times across the scheduling horizon and considering the oriented synchronization (transfer from line i to j, but not vice versa). Their model consisted of three main features: transfer event, bus bunching, and evenly spaced departures of vehicles of each line. The authors argued that in order to consider the first two elements, the objective function should maximize the number of successful transfers instead of minimizing transfer waiting time. Wu et al. (2016) advanced the work by Ibbara et al. (2012) by considering the associated transfer demand in the objective function and the existing timetable as a given input. They presented a multi-objective model to maximize the total number of passengers benefitting from synchronized transfers and minimize the maximal deviation from the departure times of the existing given timetables.

Shafahi and Khani (2014) developed a MIP model to minimize the total passenger-transfer waiting time. They used the average transfer waiting time for the total transfer demand of each line, instead of the exact value of waiting time associated with the demand of each vehicle. Fouilhoux et al. (2016) investigated the bus timetable synchronization problem with the two objectives of minimizing the passenger-transfer time as well as avoiding congestion of buses at transfer stations. The previous two studies ignored the direct effect of waiting time on passengers of individual vehicles, possibly resulting in long waiting times for many passengers. On the other hand, other researchers, (Aksu and Yilmaz (2014), Wu et al. (2015) and Liu and Ceder (2016)) considered the exact value of waiting time for passengers of each vehicle individually. Aksu and Yilmaz minimized the total passenger cost including the in-vehicle, waiting and transfer costs for all types of passengers in the system as well as the operating cost. Liu and Ceder (2016) used three decision variables in their presented model: headway of a line, departure time of the first trip, and vehicle size of each trip of lines. The objective function of their model minimized the observed vehicle load discrepancy as well as the total expected passenger waiting time. Wu et al. (2015) presented a model which minimizes the maximal passenger-waiting time through limiting the waiting time equitably over all transfer stations in a network.

In order to capture some key possible scenarios and aspects of the transfer process, we focus on the case of a single transfer node. The current study contributes to the literature by emphasizing the importance of jointly modelling of transfer waiting time, the associated demand and the vehicle capacity limitation. Also, the comparison between diverse scenarios and conditions will be presented. Although the scope of this study is restricted to a single transfer node, the outcome is expected to inform future work aimed at transfer time optimization at the network level.

3 Problem Description and Model Formulation

3.1 Problem description

Devising schedules that handle transfers optimally requires considering all directions of possible transfers and all types of time-related costs for all types of passengers—i.e., transferring passengers, in-vehicle passengers, and walk-in passengers—in a network. This approach makes transfer time optimization a complex problem not only at the network level but also at a single transfer node. As an initial step, this study is focused on formulating and optimizing the synchronization process comprehensively for a single transfer node, along with comparisons between different possible modelling approaches. Three modelling formulations were investigated in different scenarios: (1) minimize total transfer waiting time, (2) minimize total passenger-transfer waiting time, and (3) minimize total passenger-transfer waiting time with capacity limit constraint. In each scenario, three frequency combinations of the intersecting lines are chosen: (1) high and medium frequency (HM), (2) medium and low frequency (ML), and (3) low and high frequency (LH). Therefore, nine combinations are explored and compared in this study, which will be discussed in the following sections as shown in Table 2. Headways less than 10 minutes are considered short headways (high frequency), between 10 to 15 minutes are assumed medium headway, and more than 15 minutes are considered long headways (Low frequency) (Nesheli and Ceder 2015). Each model is discussed separately in the following subsections.

Table 2 Nine Modelling Approaches

Number	Scenario	Formulation	
		Transfer demand	Capacity Limit
1.1	High - Medium Frequency	---	---
2.2		√	---
3.3		√	√
2.1	Medium - Low Frequency	---	---
2.2		√	---
2.3		√	√
3.1	High - Low Frequency	---	---
3.2		√	---
3.3		√	√

3.2 Scope of the study

This study considers the transfer process among four lines (not necessarily the same two directions of one route), labelled as U, D, L and R as shown in

Fig. That is, we seek to minimize the transfer waiting time for all possible transfer directions between bus services at a transfer node (say at an intersection) consisting of four stops. Each line has its own headway and each transfer movement has a known transferring passenger demand. Each vehicle has a known number of in-vehicle passengers. The number of walk-in passengers is calculated based on the passenger arrival rate at each stop. Also, the walking time for each transfer movement can be determined based on the stop configuration and locations at the transfer node. While there are other variables affecting the transfer time such as the type of fare payment method, these are beyond of the scope of this research.

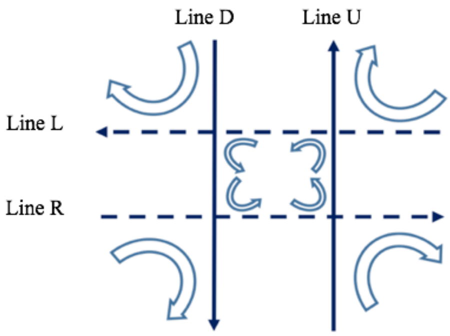

Fig. 1 Eight Possible Directions of Transfers at a single transfer node (Wu et al. 2015). D: down, U: up, L: left, and R: right.

3.3 Model description

In modelling the multi-directional transfers at a single transfer node, the model formulation must consider explicitly the arrival and departure times of vehicles on all directions of all lines and the associated types of passengers. This approach is able to capture all the possible scenarios of waiting time patterns for transferring passengers as well as in-vehicle and walk-in passengers. The model not only helps the optimization formulation but also provides a more informative demonstration of passenger transferring process at a single transfer node.

In previous studies, the decision variable mostly considered has been the departure time of the first vehicle of each line from their terminal (Shafahi and Khani 2010; Fouilhoux et al. 2016). In contrast, we define the arrival time of the first vehicle of each line at a trasnfer node in a given time period as the decsion varibale "*X*", as shown in Figure 2. Then, based on the travel time between the terminal and the transfer node, the departure time from the terminal can be determined accordingly.

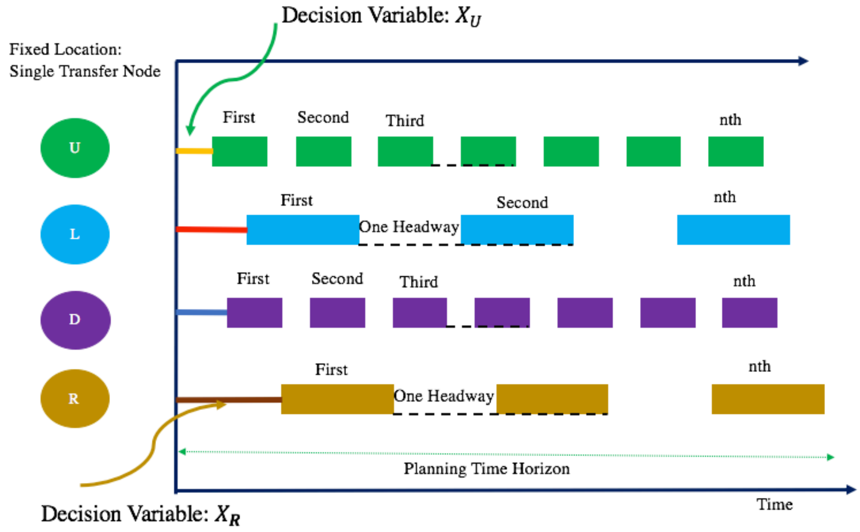

Fig. 2 The presentation of the defined decision variable. D and U represents lines with short headway and, R and L represent lines with long headway. Note: It is assumed that the long headway line has a longer dwell time compared to short headway line.

Based on the mathematical relationship between the headways, dwell times of both a feeder vehicle and a receiving vehicle and walking times, as well as the value of the decision variable for each pair, different patterns of waiting times will occur. These different patterns of waiting times indicate the need to consider the transferring demand in the objective function. If the objective function only maximizes the number of successful transfers or minimizes the total or the maximum waiting time, it is possible to end up with a solution that overemphasizes low waiting times with a very low demand, while deemphasizing high demand with longer waiting times. Therefore, the transferring demand affected by the associated waiting time must be explicitly considered. Furthermore, the capacity of the vehicles should be included in the modelling framework in order to minimize the occurrence of missing a transfer due to lack of spare capacity, while using the existing resources.

3.4 Symbol Notations

The following notation is defined and used in the model formulation.

Sets	
I	set of lines that pass the transfer node, $i \in I$
J_i	set of lines that have transfers with line i, $j \in J_i$
Parameters	
h_i	headway of line i

T	planning time horizon
P_i	Index set of feeder vehicles of line i in the planning time horizon, P_i ={1, 2, …, rounddown[T/h_i]}
P_j^+	Index set of receiving vehicles of line j in the planning time horizon, P_j^+ ={1, 2, …, rounddown[T/h_j], rounddown[T/h_j] +1}
awt_{ij}	average walking time from stop location of line i to stop location of line j
td_{ijp}	number of passengers in vehicle p of line i who want to transfer to line j
λ_j	arrival rate of walk-in passenger demand for line j (person/second)
dt_j	dwell time of vehicles of line j
ivd_{jq}	number of in-vehicle passengers of vehicle q of line j that arrive at the transfer node
ap_{ip}	total number of passengers alighting from vehicle p of line i
EC_{jq}	available capacity of vehicle q of line j
vc_{jq}	vehicle capacity of vehicle q of line j
Variables	
x_i	arrival time of the first vehicle of line i at the transfer node, main decision variable
y_{ijpq}	1: if passengers from vehicle p of line i transfer to vehicle q of line j, 0: otherwise
$arrive_{ip}$	arrival time of vehicle p of line i
$depart_{jq}$	departure time of vehicle q of line j

twt_{ijpq}	transfer waiting time for passengers from vehicle p of line i transferring to vehicle q of line j
flp_{jq}	sum of walk-in passengers and number of transferring passengers who miss their first eligible vehicle which is vehicle q of line j due to lack of enough capacity
slp_{jq}	sum of walk in passengers and number of transferring passengers who have missed their first eligible vehicle which is vehicle q of line j and also miss their second eligible vehicle due to lack of enough capacity
wp_{jq}	number of walk-in passengers for vehicle q of line j
Ttd_{jq}	total number of transferring passengers whose first possible receiving vehicle is vehicle q of line j
ED_{jq}	total demand for vehicle q of line j, including both walk-in passengers and transferring passengers whose first eligible vehicle to board is vehicle q of line j
pc	penalty cost for those who miss their second eligible vehicle, so are assumed to leave the system.

* All the time values are in seconds.

* All parameters and variables are non-negative.

3.5 Model Formulation

As explained in the above sections–our main goal is to minimize the total transfer time. In order to compare the effects of consideration of transferring demand and vehicle capacity in the model, three mixed integer programming formulations with different objective functions will be investigated along with their associated constraints.

MIP_1: Minimize the total transfer waiting time

The objective function of the first model is as below.

$$min \quad \sum_{i\in I}\sum_{j\in I_i}\sum_{p\in P_i}\sum_{q\in P_j^+} twt_{ijpq} \qquad (1)$$

The number of feeder vehicles in the planning time horizon T is equal to $P_i = \frac{T}{h_i}$ in which h_i is the headway of the feeder line. Due to the fact that, the aim is to include the transfer waiting time of passengers from all the feeder vehicles in time T, the number of receiving vehicles of line j, is equal to $P_j^+ = P_j + 1$; to guarantee availability of receiving vehicle for even the last feeder of each line. Note that the arrival time of the receiving vehicle will be out of the planning time horizon, but we will include the associated transfer waiting time if required, whose occurrence is based on the value of the decision variables x_i and x_j.

The constraints are as follows.

$$x_i \leq h_i, \qquad i \in I \tag{2}$$

$$arrive_{ip} = x_i + (p-1) * h_i \qquad i \in I, p \in P_i \tag{3}$$

$$depart_{jq} = x_j + (q-1) * h_j + dt_j, \qquad j \in I, q \in P_j^+ \tag{4}$$

$$arrive_{ip} + awt_{ij} - depart_{jq} \leq (1 - y_{ijpq}) * M_{ijpq}, \qquad i \in I, j \in J_i, p \in P_i, q \in P_j^+ \tag{5}$$

$$wt_{ijpq} \leq y_{ijpq} * h_j, \qquad i \in I, j \in J_i, p \in P_i, q \in P_j^+ \tag{6}$$

$$depart_{jq} - arrive_{ip} - awt_{ij} + M_{ijpq} * (y_{ijpq} - 1) \leq wt_{ijpq}, \qquad i \in I, j \in J_i, p \in P_i, q \in P_j^+ \tag{7}$$

$$\sum_{q \in P_j^+} y_{ijpq} \geq 1, \qquad i \in I, j \in J_i, p \in P_i, q \in P_j^+ \tag{8}$$

$$\begin{aligned} &x_i \in Z_+, && i \in I \\ &y_{ijpq} \in \{0,1\}, && i \in I, j \in J_i, p \in P_i, q \in P_j^+ \\ &arrive_{ip}, depart_{jq}, awt_{ij}, wt_{ijpq}, dt_{jq} \in R_+, && i \in I, j \in J_i, p \in P_i, q \in P_j^+ \end{aligned} \tag{9}$$

Constraint (2) ensures that the arrival time of the first vehicle of each line from the start of the panning period is less than the line's headway. Constraints (3) and (4) calculate respectively the arrival time and departure time of the following vehicles on each line based on the arrival time of the first vehicle, which is the main decision variable. The value of dt_j is considered equal to the standard average required dwell time for each line based on their headway. The three remaining constraints ensure that the model assigns the first eligible (i.e., with the least possible waiting time) receiving vehicle to each group of transferring passengers, then activate the binary variable y_{ijpq} and calculate the waiting time twt_{ijpq} accordingly. Logical constraint (5) eliminates all the receiving vehicles that depart before the arrival of each feeder vehicle. Although M can be any sufficiently big number (in both constraints (5) and (7)), we can bind it by finding the minimum possible value of the left-hand-side expression, i.e., $M_{ijpq} = \{\min_{q \in P_j^+}(depart_{jq}) - \max_{p \in P_i}(arrive_{ip}) -$

$\max_{j \in J_i} (awt_{ij})\}$, which is equal to $M_{ijpq} = -[(dt_j + (q-1)h_j) - (ph_i) - (awt_{ij})]$. In constraint (6), the h_j is actually the substitute for a big-*M*, since the longest waiting time for transferring passengers transfer to the first eligible vehicle of line j cannot be larger than the line's headway. Constraint (7) will be activated when y_{ijpq} is equal to 1, and then calculate the associated waiting time. The way that the constraints are defined, y_{ijpq} leans towards 0 in order to minimize the waiting time; however, in order to have at least one successful transfer for each feeder vehicle, constraint eight is introduced. As mentioned before, since the planning horizon is not identical for different transfer directions and we are considering a cycle for waiting time patterns, enough receiving vehicles are defined, therefore the model is always feasible.

MIP_2: Minimize the total transferring passenger-waiting time

The associated demand to each waiting time element is very critical, especially in the case of high demand transferring to a line with low frequency. Therefore, in the second model, the demand affected by each transfer waiting time is added to the objective function as below.

$$min \quad \sum_{i \in I} \sum_{j \in J_i} \sum_{p \in P_i} \sum_{q \in P_j^+} (twt_{ijpq}) * (td_{ijp}) \tag{10}$$

All of the constraints, namely (2)-(9) remain the same as the first case. However, the optimal solution may be different, since the model now considers the demand as objective coefficients and tries to assign low possible waiting times to the larger values of demand.

MIP_3: Minimize the total transferring passengers-waiting time considering vehicle capacity constraint

By considering the vehicle capacity, we need to add decision variables. The total number of alighting passengers is given to the model, but the number of boarding passengers depends on waiting times, thus need to be determined by the model. Boarding passengers include both walk-in passengers and transferring passengers. Constraint (11) calculates the number of walk-in passengers based on the arrival rate (λ) and the time interval between the departure time of receiving vehicles, where we define $departure_{j0}$= 0. Constraint (12), calculates the amount of transferring demand whose receiving vehicle is vehicle q of line j based on the value of y_{ijpq}. Then constraint (13) is introduced to provide the total number of boarding passengers for vehicle q of line j.

$$wp_{jq} \geq (departure_{jq} - departure_{j(q-1)}) * \lambda_j, \qquad j \in J_i, q \in P_j^+ \tag{11}$$

$$Ttd_{jq} = \sum_{\substack{i \in I: \\ j \in J_i}} \sum_{p \in P_i} (td_{ijp}) * (y_{ijpq}), \qquad j \in J_i \;\; q \in P_j^+ \tag{12}$$

$$ED_{jq} = Ttd_{jq} + wp_{jq}, \qquad j \in J_i \;\; q \in P_j^+ \qquad (13)$$

For the passengers who cannot board their first arriving vehicle due to lack of enough capacity, the main assumption here is that, they will only wait for the next vehicle, and if the second arriving vehicle does not have enough capacity, they would change their mode of travel and leave the system. Therefore, we introduce two new sets of decision variables: (1) flp_{jq}, the number of passengers (including both transferring passengers and walk-in passengers) who miss their first eligible vehicle, vehicle q of line j, so they will wait for the next vehicle, $(q + 1)$, for an extra h_j; their extra waiting time is the second term in the objective function, (2) slp_{jq}, the number of passengers who not only miss their first vehicle, vehicle q of line j, but also miss the following vehicle $(q + 1)$, thus leave the system. The pc is the value of penalty cost. The second group will add penalty to the system which is shown as the third element in the objective function as below.

$$\min \quad \sum_{i\in I}\sum_{j\in I_i}\sum_{p\in P_i}\sum_{q\in P_j^+}(twt_{ijpq}) * (td_{ijp}) \quad + \sum_{i\in I}\sum_{j\in I_i}\sum_{q\in P_j^+}(flp_{jq}) * (h_j)$$

$$+ \sum_{i\in I}\sum_{j\in I_i}\sum_{q\in P_j^+}(slp_{jq}) * (pc) \qquad (15)$$

Before adding new constraints, we need to define an expression for calculating the existing capacity of each arriving vehicle. Constraint (16) calculates the existing capacity as the difference between the vehicle capacity (vc_{jq}) added by the number of alighting passengers (ap_{jq}) and subtracting the number of in-vehicle passengers (ivd_{jq}).

$$EC_{jq} = vc_{jq} - ivd_{jq} + ap_{jq}, \qquad j \in J_i \;\; q \in P_j^+ \qquad (16)$$

The additional constraints are as follows:

$$flp_{jq} \geq ED_{jq} - (EC_{jq} - flp_{j(q-1)} + slp_{j(q-1)}), \qquad j \in J_i \;\; q \in P_j^+ \qquad (17)$$

$$slp_{j(q-1)} \geq flp_{j(q-1)} - EC_{j(q)}, \qquad j \in J_i \;\; q \in P_j^+ \qquad (18)$$

Constraints (17) determines the possible number of passengers who miss their first receiving vehicles, where we define $flp_{j0} = slp_{j0} = slp_{j,-1} = 0$ for all $j \in J_i$. Constraint (18) calculates the number of passengers who miss their second eligible vehicle. For example, if remaining capacity of vehicle q of line j, EC_{jq}, is 12 passengers, and the leftover passengers from vehicle $(q - 1)$, are 10 passengers, then the constraint (18) becomes redundant so the value of $slp_{j(q-1)}$, is 0 and the number of flp_{jq} is determined by the model based on constraint (17) to be equal to max$\{ED_{jq}$-2 , 0$\}$. In another example, consider remaining capacity of vehicle q of line j, EC_{jq}, is 6 passengers, and the leftover passengers from vehicle $(q-1)$, are 10 passengers, then based on constraint (18) the value of $slp_{j(q-1)}$, is 4 and therefore number of flp_{jq} is determined by the model based on constraint (17) to be equal to ED_{jq}.

4 Numerical Examples and Results

In this section the results from all the MIP models are described. All MIP models are implemented using Python 3.6.2 and solved by Gurobi 7.

4.1 Parameters and Given Data

The developed model is tested with a numerical example to describe the importance of transferring demand and capacity constraint in the problem of timetable synchronization at a transfer point. The given data and parameters value of the model are shown in Table 3, and 4. The lines' headway in the three main scenarios are presented in Table 3. To make a generic example, the assumption here is that the lines are not necessarily two directions of one route, i.e., there are mainly four different intersecting lines at a single transfer node.

Table 3 Lines' headway in different scenarios

Scenario	Headway (minutes)			
	L	U	D	R
Low - Medium Frequency (LM)	20	11	14	17
Medium - High Frequency (MH)	14	5	8	12
Low - High Frequency (HL)	18	4	9	16

The time planning period is considered 2 hours. Assuming a major single transfer node, the dwell time for vehicles of each line, dt_j, is set to be 40, 50, and 60 seconds for high, medium, and short frequency lines, respectively. The applied average walking time between the line stop's locations are shown in Table 4.

Table 4 Average walking time between stops

Stops	L – D	L – U	R – D	R – U
Time (seconds)	45	55	45	55

The penalty cost for passengers who miss their second eligible vehicle (pc), is set to the headway h_j for each line. The vehicle capacity is considered as 55 passengers for all the vehicles. The rate of walk-in passengers, (λ_i), for all the lines is considered 1 person per 1.5 minutes. The different types of passenger demand namely, in-vehicle, alighting, and transferring passengers of each vehicle of each line can be found in Appendix 1.

4.2 Results and Discussion

The three types of MIP formulations, namely MIP_1, MIP_2, and MIP_3, are tested using different headways combinations that are introduced in each scenario. The results including the optimal value of decision variables and the objective values are shown in Table 5-7.

Table 5 Results for MIP_1, minimize total transferring waiting time

Scenario	Arrival time of the first vehicle of each line (seconds)				Optimal Value for MIP_1: total transferring waiting time (seconds)	Calculated Value: total transferring passenger-waiting time (person*seconds)
	L	U	D	R		
LM	235	0	10	295	$2.5040 * 10^4$	$1.1098 * 10^5$
MH	240	55	245	720	$3.0960 * 10^4$	$1.3376 * 10^5$
LH	525	50	540	285	$3.7680 * 10^4$	$1.5955 * 10^5$

Table 6 Results for MIP_2, minimize total transferring passenger-waiting time

Scenario	Arrival time of the first vehicle of each line (seconds)				Calculated Value: total transferring waiting time (seconds)	Optimal Value for MIP_2: total transferring passenger-waiting time (person*seconds)
	L	U	D	R		
LM	485	10	0	305	$2.5200 * 10^4$	$1.0318 * 10^5$
MH	840	115	365	360	$3.1980 * 10^4$	$1.2560 * 10^5$
LH	525	50	0	525	$3.8640 * 10^4$	$1.5403 * 10^5$

Table 5 summarizes the results of all scenarios. The value of the total waiting time is determined by implementation of the optimization formulation MIP_1; however, the value of total transferring-passenger waiting time is calculated based on the optimal values of decision variables. Same calculation is applied to obtain values in table 6, however the objective function is MIP_2 and the value of total transfer waiting time is calculated accordingly. Comparing results of both methods reveals that for all the scenarios, when MIP_1 is implemented, the value of total waiting time is lower and the value of total transferring passenger-waiting time is higher compared to MIP_2. Although the saved total waiting time is greater using the objective function of minimization of total transfer waiting time, the difference is not remarkable. The improvement of total times saved by MIP_1 compared to MIP_2 are %0.63, %3.18, and %2.48 for scenarios LM, MH, and LH, respectively. On the other hand, by applying MIP_2, the saving of transferring passenger-waiting time is larger compared to MIP_1. The percentage of saved person-time are %7.02, %6.10, %3.46 for scenarios LM, MH, and LH, respectively. Therefore, using the optimal value of arrival times suggested by MIP_2, is preferable. Furthermore, by comparing the saving values between different scenarios, it is found that LM and LH scenarios are more sensitive in case of model application, compared to MH scenario. This may be because of the difficulty of synchronizing high frequency lines with medium and low frequency to obtain minimum waiting time patterns. Also, in the case of applying MIP_2 the possibility of providing larger number of passengers with lower waiting time is higher. In other words, the priority is given to the transfer direction with higher demand, which is useful especially in peak-hours when the difference between transfer demand is noticeable between directions.

Table 7 Results for MIP_3, minimize total transferring passenger- waiting time considering capacity constraint

Scenario	Arrival time of the first vehicle of each line (seconds)				Optimal Value for MIP_3: total transferring passenger- waiting time (person*seconds) with capacity constraint	Calculated Value: total transferring passenger- waiting time (person*seconds)
	L	U	D	R		
LM	485	10	0	305	$1.0318 * 10^5$	$1.0318 * 10^5$
MH	475	290	480	475	$1.2770 * 10^5$	$1.2770 * 10^5$
LH	1065	110	0	585	$1.6183 * 10^5$	$1.6183 * 10^5$

Table 7 shows that when MIP_3 is applied the value of total transferring passenger-waiting time and extra leftover passenger-waiting time, computed by the model, is equal to the calculated value of transferring passenger-waiting time. This equality means that MIP_3 offered the optimal arrival times in a way that no passenger misses their vehicles due to lack of capacity. Table 8 shows the total number of first leftover passengers, flp_{jq} for each line when MIP_2 and MIP_3 are implemented. When MIP_2 is applied, although the value of transferring passenger-waiting time is lower compared to application of MIP_3—(%0, %1.67%, and %5.06 for scenarios LM, MH, and LH, respectively)—there are 5 and 7 passengers who miss their busses due to lack of capacity in MH and LH scenario, respectively. This is because in MIP_2, the leftover passengers' penalty is not included in the objective function and it is assumed that they have successfully transferred to their first receiving vehicle, which is not correct. Therefore, as descried for scenarios MH and LH, by applying MIP_3, it is possible to (1) use the available resources optimally, (2) minimize the number of passengers who cannot board due to the capacity and (3) at the same time not add high value to the transferring passenger-waiting time. Applying capacity constraint gives more realistic value of objective functions and decision variables.

Table 8 Total number of first leftover passengers for each line

Scenario	MIP_2				MIP_3			
	2L	1U	1D	2R	2L	1U	1D	2R
LM	0	0	0	0	0	0	0	0
MH	0	0	5	0	0	0	0	0
HL	4	0	1	2	0	0	0	0

The comparison between the results of the three proposed models shows that applying objective function as minimization of total transferring passenger-waiting time along with capacity constraint is more realistic. Although in most cases the vehicle capacity is chosen separately from timetable design, in terms of transfer synchronization, it is required to consider the availability of enough capacity for successful transfer. There are some studies in this regard in which the authors optimize both line planning and timetable synchronization simultaneously by applying capacity as a decision variable (Liu and Ceder 2017). However, in our proposed model, the procedure optimizes the transfer synchronization considering the existing capacity rather than suggests additional resources. That helps to provide better fit of supply and demand.

5 Conclusion

This study was an effort not only to explore timetable synchronization at a single transfer node but also to assess the necessity and effect of different variables and constraints in model formulation. Three optimization models along with their associated constraints were introduced and tested using a numerical example: (i) minimize total transferring waiting time (ii) minimize total transferring passenger-waiting time, and (iii) minimize total transferring passenger-waiting time considering vehicle capacity constraint. The comparison between the results show two important outcomes as follows.

1. In terms of transfer synchronization, although the main goal is to minimize transferring waiting time, in the case of applying optimization methods, it is important to be cautious about the design of objective functions and constraints. Therefore, the first step is to define a performance measure clearly such as total transferring passenger-waiting time or total transfer waiting time.
2. After fixing the main objective function, it is required to capture the relations between existing variables and their associated constraints. This is crucial in order to avoid miscalculation in the objective function. For instance, in terms

of transfers, it is important to ensure successful transfers no one is missing a vehicle due to capacity limit.

As a result, for a case study, it is recommended to apply different types of modelling approaches, compare the pros and cons and then choose the best one based on the existing conditions and constraints. Specifying correct optimization formulation plays an important role in trusting and playing its outcomes.

Although the results of this study provide important insight into the importance of optimization model formulation, there are some limitations that can be investigated in future studies:

- If the intersecting lines are the two directions of the same route, it is required to add some constraints to capture the relations between the cyclic trips of vehicles.
- The extra waiting for the next eligible vehicle due to the lack of capacity is perceived differently by transferring passengers and non-transferring passengers. Therefore, it is recommended to apply different penalty cost for different types of passengers in the objective function.

6 References

1. A. Ceder, Public transit planning and operation: modeling, practice and behavior, CRC press, 2016.
2. A. Ceder, B. Golany and O. Tal, "Creating bus timetables with maximal synchronization," *Transportation Research Part A: Policy and Practice,* vol. 35, no. 10, pp. 913-928, 2001.
3. A. Eranki. "A model to create bus timetables to attain maximum synchronization considering waiting times at transfer stops.", 2004.
4. P. Fouilhoux, O. Ibarra-Rojas, S. Kedad-Sidhoum and Y. Rios-Solis, "Valid inequalities for the synchronization bus timetabling problem," *European Journal of Operational Research,* vol. 251, pp. 442-450, 2016.
5. A. Haghani and Y. Shafahi, "Bus maintenance systems and maintenance schedulig: model formulation and solutions.," *Transportation Research - Part A,* vol. 36, pp. 453-482, 2002.
6. A. Haghani, M. Banihashemi and K.-H. Chiang, "A comparative analysis of bus transit vehicle scheduling models," *Transportation Research Part B: Methodological,* vol. 37, no. 4, pp. 301-322, 2003.
7. O. J. Ibarra-Rojas, R. Giesen and Y. A.Riios-Solis, "An inegrated approach for timetabling and vehicle scheduling problems to analyze the trade-off between level of service and operating costs of transit networks," *Transportation Research Part B,* vol. 70, pp. 35-46, 2014.
8. O. J. Ibarra-Rojas and Y. A. Rios-Solis, "Synchronization of bus timetabling," *Transportation Research Part B: Methodological,* vol. 46, no. 5, pp. 599-614, 2012.
9. O. Ibarra-Rojas, F. Lopez-Irarragorri and Y. A. Rios-Solis, "Multiperiod bus timetabling," *Transportation Science,* vol. 50, no. 3, pp. 805-822, 2015.
10. C. M. Kwan, and C.S. Chang. "Timetable synchronization of mass rapid transit system using multiobjective evolutionary approach." *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)* vol.38, pp.636-648, 2008.
11. T. Liu, and A. Ceder. "Integrated public transport timetable synchronization and vehicle scheduling with demand assignment: A bi-objective bi-level model using deficit function approach." *Transportation Research Part B: Methodological,* pp.1-21, 2017.
12. T. Liu, and A. Ceder. "Synchronization of Public Transport Timetabling with Multiple Vehicle Types." *Transportation Research Record: Journal of the Transportation Research Board* 2539, pp.84-93, 2016.
13. M. Mahmoodi Nesheli, A. (. Ceder and T. Liu, "A Robust, Tactic-Based, Real-Time Framework for Public Transport Transfer Synchronization," in *21st International Symposium on Transportation and Traffic Theory*, 2015.
14. H. L. Petersen, A. Larsen, O. B. G. Madsen, B. Petersen and S. Ropke, "The Simultaneous Vehicle Scheduling and Passenger Service Problem," *Transportation Science,* vol. 47, no. 4, pp. 603-616, 2012.
15. Y. Shafahi and A. Khani, "A practical model for transfer optimization in a transit network: Model formulation and solutions," *Transportation Research Part A,* vol. 44, pp. 377-389, 2010.
16. D. Tuzun Aksu, and S. Yılmaz. "Transit coordination with heterogeneous headways." *Transportation Planning and Technology.* vol.37, no.5, pp.450-465, 2014.

17. X. Tian, and H. Niu, "A dynamic programming approach to synchronize train timetables." *Advances in Mechanical Engineering* 9.6, 2017.
18. Y. Wu, H. Yang, J. Tang and Y. Yu, "Multi-objective re-synchronization of bus timetable: Model, compexity and solution," *Transportation Research Part C: Emerging Technologies,* vol. 67, pp. 149-168, 2016.
19. J. Wu, M. Liu, H. Sun, T. Li, Z. Gao and D. Z. Wang, "Equity-based timetable synchronization optimization in urban subway network," *Transportation Research Part C: Emerging Technologies,* vol. 51, pp. 1-18, 2015.

Appendix 1

Based on the headway and the number vehicles of each line in the planning time horizon, the needed values for different types of demand are used form Table 4 to 6.

Table 9 In-Vehicle Passengers of Vehicles of each Line

	Trips									
Line	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2L	37	42	24	36	27	25	41	25	45	44
1U	45	34	45	33	27	31	37	30	44	40
1D	24	43	25	45	37	41	27	31	40	35
2R	35	26	41	38	30	29	35	37	24	41
Line	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
2L	36	21	26	45	35	39	28	33	43	35
1U	42	23	35	39	24	29	32	40	39	45
1D	29	38	45	29	34	43	37	30	29	41
2R	33	29	37	24	40	31	29	41	35	27
Line	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
2L	21	37	36	43	23	29	37	25	43	38
1U	26	43	30	39	39	21	45	30	39	29
1D	45	34	26	39	34	23	39	31	29	35
2R	35	29	41	35	30	42	25	37	33	45

Table 10 Alighting Passengers from Vehicles of each Line

	Trips									
Line	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2L	18	28	15	17	19	15	20	13	25	20
1U	17	11	21	13	9	11	14	11	26	15
1D	10	24	17	11	20	15	26	10	11	15
2R	17	13	27	24	14	15	17	18	13	20
Line	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
2L	18	23	15	21	17	18	18	15	22	20
1U	25	10	13	22	11	10	12	20	15	17
1D	13	10	15	17	12	15	15	15	11	25
2R	17	15	28	13	20	17	14	23	19	15
Line	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
2L	10	15	17	21	8	10	17	11	22	15
1U	11	20	14	16	19	7	25	12	15	10
1D	22	14	8	18	13	8	14	10	9	18
2R	12	10	21	12	13	19	9	11	12	22

Table 11 Transferring Demand from each Vehicle of each Line to the other Lines

	Trips									
Line	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2L – 1U	7	9	5	4	8	4	10	4	9	7
2L – 1D	8	10	5	9	8	8	6	6	10	7
1U – 2L	3	3	5	3	3	3	4	2	6	3
1U – 2R	3	2	6	4	2	3	2	2	6	3
1D – 2L	2	6	3	3	5	4	7	2	2	4
1D – 2R	2	5	3	2	6	2	8	2	2	4
2R – 1U	5	4	8	7	5	6	4	8	5	10
2R – 1D	5	6	10	8	4	4	4	7	5	7

	Trips									
Line	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
2L – 1U	5	8	5	9	8	8	6	4	7	5
2L – 1D	8	10	7	9	6	4	6	5	8	5
1U – 2L	6	2	3	7	2	3	2	7	4	4
1U – 2R	8	1	2	6	2	4	3	6	4	3
1D – 2L	3	2	3	5	3	4	3	3	2	8
1D – 2R	4	1	3	3	2	4	5	3	3	5
2R – 1U	7	5	9	5	6	5	3	8	8	5
2R – 1D	8	7	10	5	9	5	5	9	6	4

	Trips									
Line	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
2L – 1U	3	4	6	7	2	3	5	3	8	5
2L – 1D	2	5	5	7	3	2	7	3	7	6
1U – 2L	3	6	4	5	8	2	8	4	6	3
1U – 2R	2	8	6	5	7	2	9	6	6	4
1D – 2L	9	5	2	5	6	3	5	4	2	5
1D – 2R	7	6	2	7	4	2	5	2	2	7
2R – 1U	4	3	7	4	5	6	3	3	3	7
2R – 1D	4	2	9	5	3	7	2	3	5	9