Real-time Transit Control Using Markov Decision Process: A Case Study for Transfer Coordination

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Abstract
Transfer coordination has been a topic of research for many years, where primarily schedule adjustments have been developed to increase the number of synchronized transfers or decrease transfer waiting times. In this paper, we tackle the problem from real-time control perspective, considering stochasticity in transit travel times. First, we model a transit route by a Markov process where states are defined by the arrival time of the vehicle to each station relative to the scheduled time. Transition probabilities are developed to model stochastic link travel times conditional on the last state, which satisfy Markovian property. A Markov Decision Process (MDP) is proposed to determine optimal vehicle holding time at each stop and under each state in order to minimize total passenger times on the route. The holding can be used for applications such as maintaining schedule adherence or optimizing transfers on the route. A real case study on Light Rail Transit (LRT) lines from Twin Cities MN is presented, and optimal holding times and transfer offset times are calculated.

Keywords: Transit Operations, Real-time Control, Transfer Coordination, Holding Time, Stochastic Travel Time, Markov Decision Process

1 Introduction
Real-time operation of transit systems has been a topic for researchers for decades. Problems such as schedule adherence and transfer coordination, as well as dealing with crowding are some of the most important problems in this category [1,2]. Schedule adherence and transfer coordination, in particular, can be addressed by strategies such as vehicle holding and stop skipping. This is beyond the optimization of schedule in the service planning stage [3, 4], and is aimed to deal with the stochasticity in transit operations. Several recent studies address the problem with tactical decisions, while still using mathematical programming for real-time application [5–7]. In general, mathematical programming can be computationally more expensive than to be applied in real-time, especially when decision variables are integer.

In this paper, we study the transfer coordination problem with stochastic transit travel times by developing optimal holding times at stops along the route. Assuming that coordi-
nated transfer is a higher priority objective than on-board passengers’ delay, transit vehicles can be held at some stops before transfer stops such that it arrives to the transfer stop about the same time as the connecting route. To avoid the complexity of using mathematical programming for holding time optimization in real time, we model the transit route as a Markov process [8, 9] with transition probabilities being used to predict the arrival time of the vehicle to future stops. The goal is to develop vehicle holding policies to be applied in real time according to the actual arrival time of the vehicle at each stop. In other words, the problem is formulated as a Markov Decision Process (MDP) in which control actions are selected from a lookup table in real-time.

The paper is organized in five sections. After the introduction in Section 1, literature review on relevant topics is provided in Section 2. In Section 3, a Markov process is presented for modeling a transit route and a MDP is proposed for optimizing holding time. Section 4 presents results of numerical tests on a Light Rail Transit network. Finally, conclusions and future work are presented in Section 5.

2 Literature Review

In transit literature, real-time operation has been studied from different perspective, but two important problems are vehicle holding problem and transfer coordination. Therefore, we briefly review these topic below.

Vehicle holding problem on simple route structures was studied since 1970s [10, 11] and later on more general transit routes [12, 13]. These studies developed analytic models that take into account stochastic vehicle arrival and random passenger arrival to minimize a cost function comprising passenger waiting time and onboard passenger delays. In particular, Hickman (2001) proposed a quadratic programming model that finds optimal headway at each station for total minimum passenger time [13]. A unique aspect of the model was to assume partial information through automatic vehicle location and automatic passenger count systems. More recently, Delgado et al. (2009) proposed quadratic programming model to find optimal holding and limited boarding enforcement to minimize total passenger times [14]. Another important work, although not necessarily on holding, is by Bartholdi (2012) aiming to keep headways as uniform as possible [15]. While most of the literature on vehicle holding is on routes operating based on headway, Bartholdi (2012) used virtual schedules and aimed to keep buses running close to schedule. They used a Markov model to develop the control strategy.

Literature on transfer synchronization is also broad. A major contribution in this area was the work by Ceder et al. (2001), in which maximum transfer synchronization is sought [3]. Transfer synchronization was defined as the transfer with exact arrival time of two buses at the transfer station. They proposed a mathematical programming model with integer variables and also proposed a heuristic algorithm to solve it. Shafahi and Khani (2010) developed an extension of the model in which the sum of passengers transfer times was minimized [4]. The motivation was that, if buses arrive within a few minutes, transfers in one direction will be still seamless and even more reliable, while in the other direction passengers have to wait for nearly one more headway, and this pattern continues for future
vehicles of the same routes. Therefore, the system may still benefit from coordinated but not necessarily synchronized transfers. Another aspect of the study was to develop holding times at intermediate stops such that more transfers are coordinated. A mathematical model and genetic algorithm was developed to solve the problem in [4]. A few other papers were published on the topic, mostly built on the previous work [16–18]. A somewhat different and recent way of approaching the transfer coordination problem was through the use of tactical strategies [5, 6]. In these studies, mathematical programming models were developed for finding optimal holding time, stop skipping, and short turn decisions, in order to maximize synchronized transfers. Even in these recent studies, still mathematical programming models with integer variables were used.

Although both reviewed problems have quite rich literature, we found limited studies that address the problems from real-time perspective, or the existing methods are not efficient for real-time applications. Particularly, more research could be done when routes are operated according to schedule, have stochastic travel times, and real time control is desired for transfer coordination. Moreover, no study in the literature seem to use MDP to solve the aforementioned problem, although Markov processes are quite widely used to model similar problems [8, 9, 11, 19]. Therefore, the objective of the study is to develop real-time control policies to maximize transfer coordination (minimize passenger times) when travel times are stochastic. We modeled the problem as a MDP which is new for this particular type of problem.

3 Methodology

3.1 Markov Process Model for a Transit Route

A transit vehicle trip is given with stops \( i = 1, 2, ..., N \) and corresponding scheduled times \( \hat{t}_i \). Travel times between consecutive stops \( i \) and \( i + 1 \) are given as random variables \( \tau_{i,i+1} \) and follow a discrete distribution. The transit vehicle trip is modeled as a Markov process with states \( s \in S_i \) at transit stops \( i \) where \( S_i \) is the set of possible states at stop \( i \). For instance, if arrival times are discretized to one-minute intervals, each interval will represent a state. The process is Markovian if travel times \( \tau_{i,i+1} \) and \( \tau_{j,j+1} \) for \( i \neq j \) are independent random variables. In other words, travel time \( \tau_{i,i+1} \) is Markovian if it depends on the previous state \( s_i \) only. Therefore, the expected arrival time of the vehicle at stop \( i \), i.e. \( E(t_i) \), can be calculated according to its state at stop \( i - 1 \) and the travel time distribution from \( i - 1 \) to \( i \):

\[
E(t_i|s_{i-1}') = \sum_{s_i \in S_i} [t(s_{i-1}') + p(s_i|s_{i-1}') \tau(s_{i-1}', s_i)]
\] (1)

In a similar way, the probability of the vehicle being in state \( s \) at stop \( i \), i.e. \( P(s_i) \), is calculated using the state at stop \( i - 1 \) and transition probabilities from \( i - 1 \) to \( i \):

\[
P(s_i) = \sum_{s_i' \in S_{i-1}} P(s_{i-1}')p(s_i', s_i)
\] (2)

Equation (1) and (2) are recursive meaning that \( t(s_{i-1}') \) and \( P(s_{i-1}') \) can be expressed in terms of states at stop \( i - 2 \) and so on until the first stop of the route. Using the proposed
Markov process and knowing the state of the vehicle at a current stop, one can project the expected arrival time of the vehicle to subsequent stops.

3.2 Optimal Vehicle Holding Policy using MDP

We aim to model the real time control of a transit vehicle using the proposed Markov process. The process involves making decisions, e.g. about holding the vehicle, at each stage of operations depending on the state of the vehicle, such that an overall cost function is minimized. This is more flexible than making decisions in the planning stage and following them in real time as uncertainty in operations may cause planned decisions being suboptimal. The decisions in this case are called policies because they depend on the state of the vehicle and can change from time to time. The cost function is the total time of passengers on the route, including their waiting time and in-vehicle time.

Let’s define nonzero decision variables $h(s_i)$ representing the amount of holding time applied to the vehicle at stop $i$ when its state is $s_i$. The goal is to determine the optimal holding time such that the total passenger time is minimized. Let $x_i$ be the number of on-board passengers before departure from stop $i$ and $y_{i+1}$ be the number of passengers waiting at subsequent stop $i+1$, both deterministic values (relaxation of $y$ to random variables makes the model more realistic, which is a future research topic). By applying holding time $h(s'_i)$ to the vehicle before departure from stop $i$, the following equation will represent the total waiting time of passengers on the route:

$$C_i(s_i, h(s_i)) = h(s_i)x_i + E[w(s'_{i+1})y_{i+1} + C_{i+1}(s'_{i+1})]$$

(3)

where the first term is the on-board passengers delay, $w(s'_{i+1})$ is the waiting time of a passenger at stop $i + 1$ if the transit vehicle arrives at state $s'$, calculated from the following equation:

$$w(s'_{i+1}) = \max\{0, t(s'_{i+1}) - \hat{t}_{i+1}\}$$

(4)

and $C_{i+1}(s'_{i+1})$ is the expected cost at state $s'_{i+1}$ of the subsequent stop determined recursively in future stages. Using transition probabilities, the total cost of applying holding time $h(s_i)$ at state $s$ of stop $i$ is determined as following:

$$C_i(s_i, h(s_i)) = h(s_i)x_i + \sum_{s' \in S_{i+1}} p(s''_i, s'_{i+1})(w(s'_{i+1})y_{i+1} + C_{i+1}(s'_{i+1}))$$

(5)

where $s''_i$ is the state of the vehicle at stop $i$ after applying holding time $h(s_i)$ to state $s_i$. In particular, if states are defined as delay relative to the scheduled time, then $s''_i = s'_i + h(s_i)$.

In order to minimize total passenger times along the route, the cost is broken to small components such that the action at each station will impact the cost in that station plus the state of the vehicle at future stops. The optimal holding time $h$ can be determined at each stop and be applied in real time according to the state of the vehicle. The cost function at
each stop is optimized using the following single-variate mathematical programming:

$$\min_{h(s_i)} C_i(s_i, h(s_i))$$ (6)

subject to

$$t(s_i) + h(s_i) \geq \hat{t}_i$$ (7)

$$0 \leq h(s_i) \leq \bar{h}$$ (8)

The objective function represents the total delay of passengers on the route, as explained in equation (5). Constraint (7) guarantees that the vehicle is held enough until the scheduled time. In other words, vehicles may not depart from a stop earlier than their scheduled time, otherwise, passengers will miss the vehicle if they coordinate their departure time with schedule. This is a common practice at timepoint stops and may not be necessarily enforced at all stops. Constraint (8) guarantees that holding time does not exceed a threshold $\bar{h}$ due to passengers perception of delay. Note that constraints (7) and (8) could make the problem infeasible if vehicle is more than $\bar{h}$ units of time earlier than scheduled time. An infeasible case would indeed suggest schedule update in the planning stage because the scheduled time $\hat{t}_i$ is not designed appropriately. Therefore, this case is not considered in this real-time holding problem. A simple solution for the infeasible case would be to relax constraint (8) and if in the solution $t(s_i) + h(s_i)$ is less than $\hat{t}_i$, override it with $\hat{t}_i$.

We acknowledge that vehicle holding in this case will likely return zero holding time if there is not a significant benefit further along the route. Equation (5) written for the last stop determines the cost of the terminating state $C_N$ at the end of the route and if the cost is zero, the optimal operations will be not to hold the vehicle unless there is travel time saving by departing at a later time. This travel time saving would mean Non-FIFO link travel times which is unlikely in traffic networks and almost impossible in rail systems. Therefore, a more realistic application of holding is presented in the next subsection, where holding can benefit transfer passengers by reducing their waiting time at transfer stations.

### 3.3 Transfer Coordination using MDP

A more general version of vehicle holding problem is for coordinating transfers between a pair of routes. This is the case where two routes intersect and passengers transfer from one route to the other, thus waiting for some time at the transfer stop. A timed transfer would be ideal in which vehicles on the two routes arrive to the transfer stop about the same time and passengers complete the transfer seamlessly (with near zero waiting time). Vehicle holding on one (or both) routes could be beneficial in this case because vehicles’ operations can be monitored and controlled by holding at some stops such that the two vehicles arrive to the transfer stop at the same time. So while holding increases the delay of onboard passengers, it can reduce waiting time at transfer stations, which is usually more onerous. Moreover, a seamless transfer may make positive impression on users, as a sign of reliable service.

Assuming routes $R_1$ and $R_2$ intersect at a transfer stop represented by $j_1$ on route $R_1$ and $j_2$ on route $R_2$, the holding problem for each route can be extended such that the sum of on-board passengers’ delay and the transferring passengers’ delay is minimized. The system’s state is defined by a tuple $s_i = (s^1_i, s^2_i)$ as the combined states for the two routes at stage $i$. The decision variable is extended to a tuple $h(s_i) = (h^1(s^1_i), h^2(s^2_i))$ and the system cost is
defined as below:

\[ C_i(s_i, h(s_i)) = h(s_i) \cdot x_i + E(w(s'_{i+1})y_{i+1} + C_{i+1}(s'_{i+1})) \]  

(9)

where bold notation represents the tuple of the parameters for routes \( R_1 \) and \( R_2 \). Similar to the holding problem, the first term in equation (9) is the holding delay for on-board passengers, the second term is the delay for waiting passengers at the subsequent stop, and the last term is the cost for the remaining part of the routes. A key point in the transfer coordination problem is the cost at the terminating state, which is defined to be the transfer stop \( j \). Let’s assume that the number of passengers wanting to transfer from route \( R_1 \) to route \( R_2 \) and vice versa are represented by \( z_1 \) and \( z_2 \) respectively. Also, let \( H_1 \) and \( H_2 \) be the headway for routes \( R_1 \) and \( R_2 \) respectively. The system cost at termination for a given system state \( s_j \) is given as below:

\[
C_j(s_j) = \begin{cases} 
0 & ; t(s_1) = t(s_2) \\
z_1[t(s_2^j) - t(s_1^j)] + z_2[t(s_1^j) - t(s_2^j)] & ; t(s_1^j) > t(s_2^j) \\
z_1[t(s_2^j) - t(s_1^j)] + z_2[t(s_1^j) - t(s_2^j) + H_1] & ; t(s_1^j) < t(s_2^j) 
\end{cases}
\]

(10)

Equation (10) indicates that, if the vehicles in two routes arrive to the transfer stop at the same time, no transfer passenger has to wait, so the cost is zero. But if one of the vehicles arrive earlier, passengers from that vehicle have to wait for the difference in the vehicles’ arrival times, and passengers from the second vehicle have to wait one headway minus the time difference for the next vehicle in the connecting route. If there are multiple transfer stops on a route, transfer waiting times will be calculated and added to the system costs (9) in a similar way.

3.4 Solution Algorithm

The complexity of the problem lies in its stochastic nature. However, the decomposition of cost to Bellman’s equations (3) or (9) allows for efficient solution algorithms based on stochastic dynamic programming and iterative optimization of the cost at each stop. A solution algorithm based on value iteration is proposed in Figure 1. In the initialization step, cost at termination stop is determined and at every other state is set to infinity. Going backward in direction of the route, at each stop the optimal holding time is determined such that to minimize the sum of the immediate cost of holding and the expected cost at the next stop. The optimization process continues until all the stops are processed. Note that unlike standard implementation of value iteration, an outer loop to iterate and update cost values is not required because stops are processed from last to one. Therefore, the presented algorithm is a one-shot value iteration, which is indeed sufficient for optimal solution.

An algorithm for the transfer coordination problem can be developed similar to Algorithm 1, with the difference that cost initialization is done according to equation (10), and cost updating is done according to equation (9).
Algorithm 1

Inputs: \( p, t, \hat{t}, x, y \)
Outputs: \( h \)

\[
C(s_i) = \infty; \forall s \in S_i; \forall i \in I \backslash \{N\}
\]
\[
C(s_N) = x_N[t(s_N) - \hat{t}_N]^{+}; \forall s \in S_N
\]

For \( i \in I \) in reverse order:

For \( s' \in S_i \):

\[
C(s'_i) = \min_{h(s'_i)} (h(s'_i)x_i + \sum_{s \in S_{i+1}} p(s_{i+1}|s'_i)[(t(s_{i+1}) - \hat{t}_{i+1}]^{+}y_{i+1} + C(s_{i+1}))
\]

\[
h^*(s'_i) = \arg \min_{h(s'_i)} (h(s'_i)x_i + \sum_{s \in S_{i+1}} p(s_{i+1}|s'_i)[(t(s_{i+1}) - \hat{t}_{i+1}]^{+}y_{i+1} + C(s_{i+1}))
\]

Figure 1: Value iteration algorithm for optimal transit vehicle holding problem

4 Numerical Tests

The motivation of this research was to improve the quality of transfers in a Light Rail Transit system, so the proposed method is applied to a real case study and results are presented.

4.1 Markov Process Model for a Light Rail Transit

The test network is a set of two LRT lines in the Twin Cities MN region. Figure 2 shows the layout of the two routes, Metro Green Line in East-West direction connecting downtown Saint Paul to Downtown Minneapolis through the University of Minnesota campus, and Metro Blue Line in North-South direction connection downtown Minneapolis to Minneapolis-St. Paul International Airport and continuing further south including service to Mall of America. An arterial Bus Rapid Transit operates in conjunction with the two LRT routes and more regular bus routes. The two LRT routes overlap on a segment in Downtown Minneapolis, and split after US Bank Stadium Station. Therefore, the main transfers happen by passengers traveling between St. Paul and Southern parts of the region. Particularly, we consider a directional trips starting with Green Line going Westbound, and transferring to Blue line Southbound, which represents a major commute pattern in the afternoon peak period. A seamless transfer at the transfer station will benefit this group of passengers.

Ridership data has become available through Automated Passenger Counts from Metro Transit. Green Line’s boardings, alightings and loads from the beginning of the route through the transfer stop are shown in Figure 3. Station 19 is the transfer stop, showing relatively high alighting volume per train. It is assumed in this paper that 90 percent of this alighting volume transfers to Blue Line in the southbound direction and the remaining 10 percent egresses to nearby destinations.

Train arrival time data at stations has become available through Automated Vehicle
Location data from Metro Transit. The arrival data was processed to determine the states and transitions between states. States are defined to be the number of minutes late or early relative to scheduled time, with negative sign representing early and positive sign representing late. It was observed that states ranging from -2 to +5 cover most cases, and any realization outside this range was mapped to a nearest state (e.g. 4.5 minutes early is considered in the 2 minutes early state). Figure 4a shows the state probabilities for the Green Line aggregated over all the stops. Station-specific distributions were calculated and used in the numerical test, but for concise visualization purpose, only the aggregate chart is shown. Transition probabilities were calculated by counting the number of occurrences for each state combinations at stops \( i \) and \( i+1 \), Aggregate transition probabilities are shown in Figure 4b, and station-specific ones were used in the numerical test. An interesting observation was that, with a 2-minute scheduled travel time between stops, it is impossible for a train to move two states backwards, but is quite common to move one state backward. For example, if the train is three minutes late, with probability of 0.27 it will be only two minutes late at the next station, catching up one minute of its delay, but it is impossible to be in state one or earlier.
4.2 Optimal Holding Time for Transfer Coordination

Both LRT lines have 10 minutes headway, with 4 minutes offset time at the transfer station. That is, Green line trains are scheduled to arrive 4 minutes before Blue Line trains. So, the transfer waiting time is calculated using the headway and offset time according to equation (10). One may presume that scheduling zero offset time leads to minimum transfer waiting time as seen in Figure 5a, but as figure 5b shows, when the arrival time of the connecting route is uncertain, synchronized transfer (i.e. zero offset time) is not optimal, and instead, in our case study, 2-3 minutes offset provides the best buffer to create a reliable transfer.

The algorithm was applied to the network, with an assumed waiting time coefficient of 2 compared with a coefficient of 1 for in-vehicle time, and a coefficient of 3 for transfer waiting time. These parameters are selected to reflect the higher disutility of waiting, especially at transfer stations. Also, a maximum holding time of 2 minutes is assumed, to make it tolerable for onboard passengers. Figure 6a shows the optimal holding time policy at each station and state. As seen, holding time is not beneficial in most of the stations until two stations before the transfer station. The positive holding times at the station preceding
to the transfer station implies that, it would be better for passengers to stay onboard the current vehicle longer and spend less time at the transfer station, which is obviously the result of larger coefficient for the latter time. One may consider a potentially negative effect of keeping passengers onboard immediately before the transfer station. To avoid this issue, transit operators can display real-time arrival information of connecting vehicles on board the trains.

Another test was conducted to evaluate the method for more than one transfer from a route. When more transfers are involved, chances are optimal decision may require complex judgment and calculation by dispatchers or operators. So we included this to shed light on potential benefits of our proposed method. For this case, transfer to the arterial BRT route A Line at station 10 is considered. It was quite interesting that the model returns positive holdings at multiple stations in the optimal policy, as shown in Figure 7. We expect the model will be more effective when multiple routes and transfers between them need to be optimized simultaneously.
A Monte Carlo simulation was used to measure the effectiveness of the developed policies. Using 10,000 trials, the average transfer waiting time when holding policies are applied was 3.0 minutes compared with 2.1 minutes in the baseline case without holding (Figure 8). This verifies that the existing schedule (4 minutes offset) works very well in practice, although for about one million transfer passengers per year, approximately 15,000 passenger-hours saving would be a considerable number, which comes with virtually no cost.

4.3 Optimal Offset Time

An extension of the test was conducted to determine the optimal offset time at the transfer station, by slightly changing the Green Line’s dispatch time from the first station. The algorithm was run for different offset times and the system cost was calculated to find the minimum value. Figure 9 shows the system cost for different offset times, where an offset of 3 minutes is found optimal. It happened that with optimal offset time, no holding is required at any station. Also, the currently used offset time of 4 minutes returns a very low system
cost, confirming that the current operation is not far from optimal.

![Figure 9: Total system cost with various offset times](image)

5 Conclusions

This study adopts a Markov process for modeling a transit route with stochastic travel times, and develops a decision process to optimize holding time for minimum passenger times in the system. A particular application to transfer coordination is outlined. A dynamic programming algorithm is proposed to find optimal policies for real-time application. Results from a case study with two LRT and one BRT lines are provided, presenting expected outcomes. Holding time policies could be more beneficial when multiple transfers exist from and to a transit route, or when multiple routes are coordinated simultaneously. Test results indicate that 3 minutes transfer buffer time returns minimum passenger delay, and the current transfer buffer time of 4 minutes is quite effective.

This study can be considered as a first step towards using MDP for real-time transit control. Thus, a few simplifying assumptions were made that can be relaxed in future studies. First, we assumed that route headways are relatively large and passengers coordinate their arrival with schedule, so the number of passengers boarding at each stop is known deterministically. This can be relaxed to stochastic parameters $y_i$ such that the cost function would be determined by the expected number of passengers. Moreover, considering next vehicles in the same route and the impact of vehicle holding on the route headway would be a more realistic study. Stop skipping, viewed as opposite of holding, can be considered as another decision variable towards minimizing the system cost, although it is difficult on rail systems. Finally, multiple transit routes and multiple transfer stops on each route can be considered as an interesting application in the context of network-wide control.

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References


