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Logit-based transit assignment: Approach-based formulation and paradox revisit

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Abstract This paper proposes an approach-based transit assignment model, under the assumption of logit-based stochastic user equilibrium (SUE) with fixed demand. This model is proven to have a unique solution. A cost-averaging version of the self-regulated averaging method (SRAM) is developed to solve the proposed problem. It is proven that the algorithm converges to the model solution. Numerical examples with discussions are presented to investigate the model properties, a paradoxical phenomenon due to the stochastic nature of the model, capacity paradox, and the performance of the proposed algorithm. The proposed methodology is demonstrated to be able to solve the Winnipeg transit network.

Keywords: Transit assignment · Logit-based stochastic user equilibrium · Paradox · Approach-based formulation

1 Introduction

The transit assignment problem has received considerable attention, as finding solutions for this problem is essential for both planning, designing, controlling, or managing transit networks and evaluating transit system performance. Traditionally, transit assignment problems are either formulated as link-based models or path-based models in which the link- and path-based models adopt link and path flow as

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decision variables, respectively. The outputs of the link-based model are in terms of passenger link flows, which do not allow determining path flows easily. The path flow information is useful to determine the impact of path-specific cost (or cost-saving) for a group of passengers such as fare discounts resulting from transfers between specific transit lines. In order to overcome this problem while retaining the advantages of high computation efficiency of the link-based model, Szeto and Jiang (2014) proposed the approach-based formulation of the UE transit assignment problem; Jiang and Szeto (2016) also formulated their reliability-based stochastic transit assignment problem as an approach-based transit assignment problem. However, *extensions to SUE transit assignment have not been found*.

SUE transit assignment models are usually solved by the techniques for FP problems including the method of successive averages (MSA). In order to improve the convergence rate of MSA, Liu et al. (2009) proposed the flow-averaging version of the SRAM, which adopted varying step sizes during the solution process. Long et al. (2014) further reformed the flow-averaging SRAM into a cost-averaging version. However, the application of this cost-averaging method for solving transit assignment problems, including our approach-based problems, has not been reported in the literature.

The proposed approach-based SUE transit assignment model can be used to evaluate network design strategies and identify possible paradox occurrence. In the literature, studies have been conducted to identify and analyze the paradoxical phenomena of transit assignment problems. However, *little attention has been paid to the paradoxical phenomenon caused by the stochastic nature of SUE transit assignment. It is also unclear whether capacity paradox can still be observed under the SUE condition.*

Specifically, this paper makes the following contributions.

- It proposes an approach-based transit assignment model under logit-based stochastic user equilibrium; this model is more general than the UE counterpart; the model is proven to have a unique solution.
- It proposes to use the cost-averaging SRAM to solve the FP problem; the cost-averaging SRAM is proven to be convergent and can solve large transit networks.
- It illustrates the existence of the paradoxes caused by the stochastic nature of the model and passengers' non-cooperative behavior in transit networks in the context of stochastic user equilibrium. It shows that improving a transit route in the network or adding a new transit route to the network may not necessarily improve the performance of the transit system in terms of expected total system cost and network capacity.
- It investigates the occurrences of the two paradoxes and provides insights and suggestions on transit network design to avoid both the occurrence of these paradoxes.

2 Model Formulation

2.1 Assumptions

The following classical assumptions are made: A1) Passengers arrive at transit stops randomly. A2) A passenger waiting at a transfer node considers a set of attractive lines following the definition specified by De Cea and Fernández (1993). A3) The waiting time for a transit line on a link is independent of the waiting times for other lines on the same link. A4) Vehicle headways follow an exponential distribution. A5) Passengers' route choice behaviors are in a stochastic manner. A6) The travel demand between each origin-destination (OD) pair in the system is known and fixed. A7) The capacity of each transit vehicle is assumed to be the same. A8) The sub-network between each OD pair is acyclic.

2.2 Cost components

Three cost components are included in the expected total cost: mean in-vehicle travel time cost t_s, mean waiting time cost ω_s, and perceived congestion cost ϕ_s. For link *s*, the expected total travel cost c_s is given by

$$c_s(\mathbf{v}) = \mu_T t_s + \mu_W \omega_s + \mu_W \phi_s, \quad \forall s \in S, \tag{1}$$

where μ_T and μ_W are the corresponding values of time.

2.3 The approach-based SUE transit assignment model

Following Szeto and Jiang (2014), the approach of a node is defined by the link *coming out* from that node, and the approach probability is defined as the probability of an approach to be chosen by passengers leaving the node via that approach. The approach proportion and passengers flow of link *s* towards destination *d* can be calculated as follows:

$$L_s^d = \exp\left(\theta(\pi^{t(s)d} - \pi_s^{t(s)d})\right), \forall s \in S, d \in D, \tag{2}$$

$$W_s^d = L_s^d \left(\delta_{h(s)}^d + \sum_{m \in A_{h(s)}^+} W_m^d \right), \forall s \in S, d \in D, \tag{3}$$

$$\alpha_s^d = \frac{W_s^d}{\sum_{m \in A_{t(s)}^+} W_m^d}, \quad \forall s \in S, d \in D, \text{ and} \tag{4}$$

$$v_s^d = \alpha_s^d \left(q^{t(s)d} + \sum_{m \in A_{t(s)}^-} v_m^d \right), \quad \forall s \in S, d \in D. \tag{5}$$

where θ is the degree of passengers perception in travel cost; L_s^d, W_s^d, α_s^d and v_s^d is the likelihood, weight, approach proportion and passenger flow of passengers using link *s* traveling to destination *d* respectively; *h(s)* and *t(s)* are the head and tail nodes of link *s*; q^{rd} is the passenger demand from origin *r* to destination *d*; π^{id} is the minimum travel cost from node *i* to destination *d*; π_s^{id} is the minimum travel cost from node *i* to destination *d* using link *s*; A_i^+ (A_i^-) is the set of links coming out from (going into) node *i*; $\delta_{h(s)}^d$ =1 if *h(s)*=*d*, $\delta_{h(s)}^d$ =0 otherwise.

According to Eqs. (1) to (5), approach probabilities are functions of themselves, and the approach-based SUE problem can be formulated as an FP problem: to find $\boldsymbol{\alpha}$ such that

$$\boldsymbol{\alpha} = \mathbf{g}(\boldsymbol{\alpha}). \tag{6}$$

It can be proved that the approach-based SUE problem (6) has exactly one solution, and the solution satisfies the logit-based SUE condition.

3 Solution Algorithm: Cost-averaging SRAM

According to Eqs. (1) to (5), FP problem (6) can also be reformulated in terms of link travel cost, expressed as

$$\mathbf{c} = \mathbf{Y}(\mathbf{X}(\mathbf{c})). \tag{7}$$

The solution existence and uniqueness of the FP problem (6) also ensure the solution existence and uniqueness of the FP problem (7) as discussed by Cantarella (1997).

The descent direction denoted by **h** can be applied to solve FP (7), given by

$$\mathbf{h} = \mathbf{Y}(\mathbf{X}(\mathbf{c})) - \mathbf{c}. \tag{8}$$

The convergence of the proposed c-SRAM for the approach-based SUE transit assignment problem can also be proved similar to Cantarella’s 1997 cost-averaging algorithm.

4 Numerical Studies and Results

4.1 Paradoxical phenomena

Sheffi and Daganzo (1978) discussed the paradoxical phenomenon arises with stochastic traffic assignment models, where improving an existing route or adding a new route in a road network sometimes results in increased total system travel cost due to the stochastic nature of the problem. Jiang and Szeto (2016) also discussed the capacity paradox arises with their reliability-based user equilibrium transit assignment model, in which the network maximum throughput may be reduced after new transit lines are added to a transit network or after the frequency of an existing line increases. Similar paradoxical phenomena can also be observed in the proposed SUE transit assignment model. Following the network capacity defined by Yang and Bell (1998), the network capacity is the maximum throughput of the network at which all of the bottleneck links just reach their capacities under the equilibrium condition, and a bottleneck link is the link with the lowest capacity on a path. In order to illustrate the aforementioned paradoxical phenomena, a small sample transit network is developed, as presented in Fig. 1. As shown in Table 1, two scenarios and cases are discussed to illustrate the two aforementioned paradoxical phenomena.

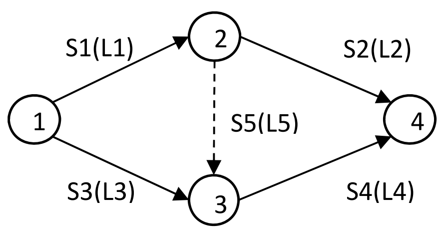

Fig. 1 Small example network

Table 1 Cases considered in each scenario

Scenario	Cases considered
Adding a new line to the network	Case 1: L5 (S5) is not provided Case 2: L5 (S5) is provided with varying frequency
Improving an existing line in the network	Case 2: L5 (S5) is provided with varying frequency

Fig. 2 shows the paradox region of the two paradox, when increment in line frequency leads to increment in expected total system cost and decrement in network throughput. Sheffi and Daganzo (1978)'s paradox is mainly caused by the stochastic nature of the problem. When a worse path (i.e., path using S5) is added or improved, the stochastic nature of the problem ensures that some passengers select the worse path, and thus the total system cost increases. On the other hand, the capacity paradox is mainly caused by passengers' non-cooperative route choice behavior. The addition/improvement of path S1-S5-S4 attracts more passengers to use it, occupying the bottleneck links in the competitive paths and reducing the total network throughput. As a result, Sheffi and Daganzo (1978)'s paradox occurs when the frequency of the additional line (i.e., L5) is low, while the network throughput paradox occurs when the frequency of the additional line is high, as shown in Fig. 2. This result implies that, when considering adding new transit lines to the network or improving frequencies of existing lines in the network, the frequencies should be carefully designed to avoid the occurrence of the two paradoxes. Addressing one paradox issue does not imply that the other paradox issue has also been addressed. A bi-objective bilevel transit network design model is needed for such purpose.

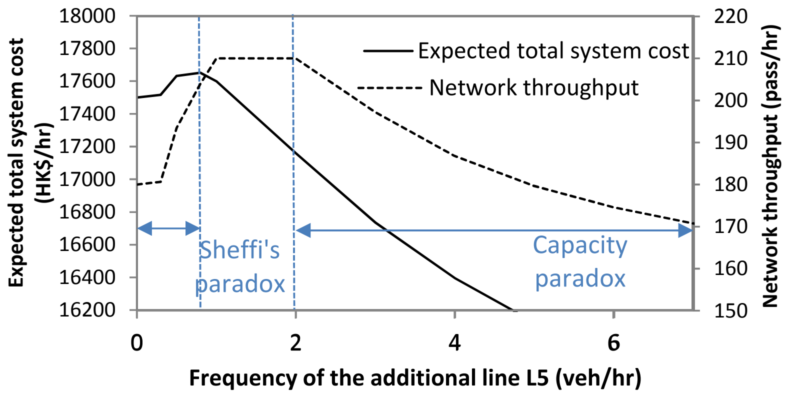

Fig. 2 Occurrence of the two paradoxes

4.2 The performance of the cost-averaging SRAM

The performance of the proposed c-SRAM is also tested using the Winnipeg transit network data provided in the EMME/4 software as shown in Fig. 3.

The length of descent direction $\left\|\mathbf{h}^k\right\|$ at each iteration in the c-SRAM given by

$$\left\|\mathbf{h}^k\right\| = \sqrt{\sum_{s\in S}\left(c_s^k - c_s^{k-1}\right)^2}, \quad \forall k > 1, \tag{9}$$

as well as the difference in expected total system cost between two adjacent iterations at each iteration are shown in Fig. 4. As shown in the figure, the proposed c-SRAM converges efficiently and is capable of solving large-scale transit networks.

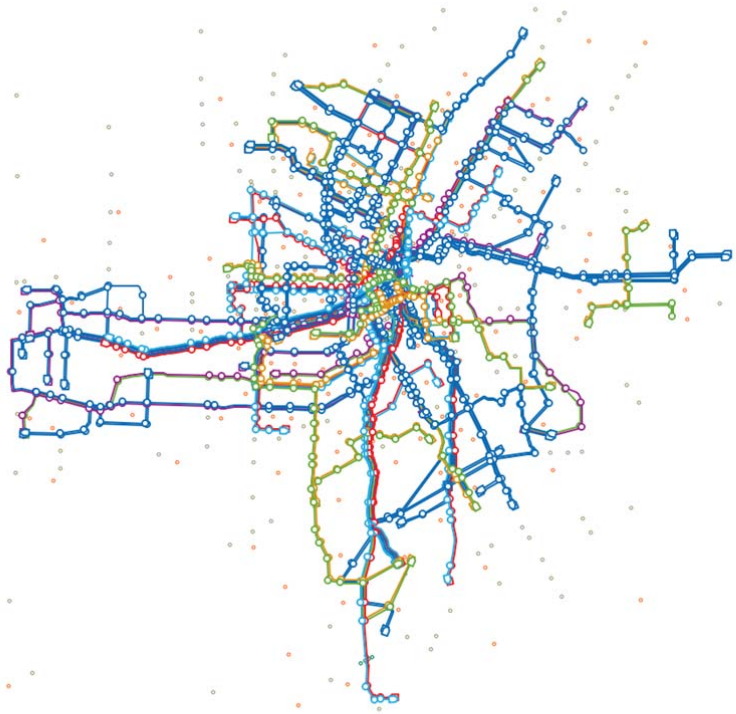

Fig. 3 The Winnipeg transit network

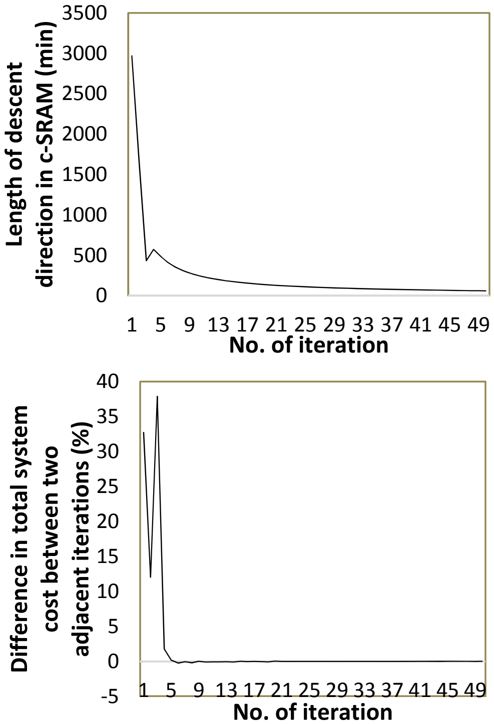

Fig. 4 Convergence of the c-SRAM for solving the Winnipeg network

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References

Cantarella, G. E., 1997. A general fixed-point approach to multimode multi-user equilibrium assignment with elastic demand. *Transportation Science, 31*(2), pp. 107–128.

De Cea, J. and Fernández, E., 1993. Transit assignment for congested public transport systems: an equilibrium model. *Transportation Science, 27*(2), pp. 133–147.

Jiang, Y. and Szeto, W. Y., 2016. Reliability-based stochastic transit assignment: Formulations and capacity paradox. *Transportation Research Part B: Methodological, 93*, pp. 181–206.

Liu, H. X., He, X. and He, B., 2009. Method of successive weighted averages (MSWA) and self-regulated averaging schemes for solving stochastic user equilibrium problem. *Networks and Spatial Economics*, *9*(4), pp. 485–503.

Long, J. C., Szeto, W. Y. and Huang, H. J., 2014. A bi-objective turning restriction design problem in urban road networks. *European Journal of Operational Research*, *237*(2), pp. 426–439.

Sheffi, Y. and Daganzo, C. F., 1978. Another “paradox” of traffic flow. *Transportation Research*, *12*(1), pp. 43–46.

Szeto, W. Y. and Jiang, Y., 2014. Transit assignment: Approach-based formulation, extragradient method, and paradox. *Transportation Research Part B: Methodological*, *62*, pp. 51–76.

Yang, H. and Bell, M. G., 1998. A capacity paradox in network design and how to avoid it. *Transportation Research Part A: Policy and Practice*, *32*(7), pp. 539–545.