Estimation of Denied Boarding in Urban Rail Systems: Alternative Formulations and Comparative Analysis

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Introduction

Increases in ridership are outpacing capacity in a number of urban rail transit systems, including Hong Kong’s Mass Transit Railway (MTR), the London Underground, and the New York subway system (Zhu et al., 2017). Crowding at stations and on trains is a concern due to its impact on safety, service quality, and operating efficiency. Various studies have measured passengers’ willingness to pay for less crowded conditions (Li and Hensher, 2011) and suggested incorporating the crowding disutility in investment appraisal (Haywood and Koning, 2015). Given the interest in dealing with crowding-related problems, developing related measures of performance is important. The number of times passengers are denied boarding and how long they wait before they can board a later train are often used as performance metrics.

However, the problem of determining the number of times a passenger is denied boarding is not trivial. While some agencies conduct manual counts to collect this data, inevitably the results are based on small sample sizes, are subject to measurement error, and are too costly to rely on. Since it is not directly observable with current automated data sources, various approaches have been proposed to estimate it from smart card (AFC) and train movement (AVL) data. These methods belong to two broad categories: a) statistical (inference and regression models); and b) assignment. Table 1 summarizes their main characteristics.

Table 1: Comparison of existing methods for denied boarding estimation

<table>
<thead>
<tr>
<th>Approach</th>
<th>Data</th>
<th>Level</th>
<th>Applications</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical inference</td>
<td>AFC (tap-in and out)</td>
<td>Station</td>
<td>Performance measurement</td>
<td>Needs access/egress time distributions</td>
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<td></td>
<td>AVL</td>
<td></td>
<td></td>
<td>Sensitive to model parameters</td>
</tr>
<tr>
<td></td>
<td>Walk distance/speed</td>
<td></td>
<td></td>
<td>Unsupervised learning</td>
</tr>
<tr>
<td>Regression</td>
<td>AFC (tap-in)</td>
<td>Station</td>
<td>Performance measurement</td>
<td>Requires observations of denied boarding for calibration (supervised learning)</td>
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<tr>
<td></td>
<td>AVL</td>
<td></td>
<td>Prediction</td>
<td></td>
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<td></td>
<td>Denied boarding observations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Network Assignment</td>
<td>OD flows</td>
<td>Network</td>
<td>Performance measurement</td>
<td>Applied at the network level</td>
</tr>
<tr>
<td></td>
<td>Path choice fractions</td>
<td></td>
<td>Planning</td>
<td>Various crowding metrics</td>
</tr>
<tr>
<td></td>
<td>AVL</td>
<td></td>
<td></td>
<td>Requires capacity</td>
</tr>
<tr>
<td></td>
<td>Capacity</td>
<td></td>
<td></td>
<td>Deterministic</td>
</tr>
</tbody>
</table>

The statistical methods are either based on unsupervised learning or use observations for calibration. Zhu et al. (2017) formulate the problem of estimating the probability density function of the number of times a passenger is denied boarding as a maximum likelihood problem of observing individual journey times in the sample, given pre-determined distributions of walking times (or speeds), and AVL and AFC data from closed systems (tap-in and tap-out transactions). The main limitations are the need for access/egress time distributions and the sensitivity to input parameters. Miller et al. (2018) proposed a regression model that uses passenger arrivals and train
departure times from AFC (tap-in only) and AVL data to estimate denied boardings. The method works at the station level and requires denied boarding observations for calibration.

Assignment methods are mostly used for planning applications. However, in principle, they can also be used to estimate denied boarding using actual (as opposed to scheduled) train departures and arrivals at stations (Stasko et al., 2016). Schedule-based assignment (Nuzzolo et al., 2012) is appropriate for this purpose. However, such approaches require assumptions about (deterministic) capacity, and route choice fractions.

The paper focuses on developing inference methods to relax some of the limitations of existing approaches using AFC (tap-in and tap-out) and AVL data. The problem is modelled using a mixture distribution framework that incorporates a priori structural information. It is data-driven and does not require information from actual observations, or assumptions about access/egress time distributions. The paper also presents an event-based deterministic transit assignment algorithm with explicit capacity constraints and priority boarding. A case study illustrates the application of the proposed methods using actual and synthetic data and compares them against existing methods.

Methodology

A. Mixture Model Inference

Inference methods, in general, are based on the hypothesis that (observed) journey time distributions can be used to estimate the underlying denied boarding probabilities after controlling for (estimated) access/egress times and (observed) in-vehicle times. The proposed method treats the problem explicitly as a clustering problem. It assumes that passengers on a given OD pair, traveling within the same (short) time period, belong to different clusters, with cluster $k$ representing the passengers who board the $k^{th}$ train after arriving on the platform.

We assume that the observations include a set of passengers on a given OD pair that travel during the same time period (e.g. within a 15 minute interval). Given a passenger’s tap-in time $t^{\text{in}}$, and tap-out time $t^{\text{out}}$, the passenger’s journey time is $t = t^{\text{out}} - t^{\text{in}}$. Let $k$ be the (unknown) cluster the passenger belongs to (the train the passenger boarded after his/her arrival on the platform). Assuming that the journey time, conditional on $k$, follows a Gaussian distribution $\mathcal{N} (\mu_k, \sigma_k)$ with parameters $\mu_k$ and $\sigma_k$, the probability of observing the journey time $t$ is given by:

$$p(t; \Omega, \Sigma) = \sum_{k=1}^{K} \omega_k \mathcal{N} (t; \mu_k, \sigma_k)$$

(1)

Where, $p(t; \Omega, \Sigma)$ is the overall probability density function (pdf) of the journey time $t$; $\omega_k$ the (unknown) fraction of passengers in group $k$, $\omega_k > 0$, $\sum_i \omega_k = 1$.

Figure 1 illustrates the denied boarding estimation framework, where $T$ is the time period; $s$ the hidden state; $\Omega$ and $\Sigma$ unknown parameters; the red arrows indicate relationships between groups; $G_i - G_k$ are the boarding groups. The method requires AFC and AVL data for a given OD pair (assuming no transfers and no route choice). The origin represents the station of interest for which denied boarding metrics will be estimated.
Figure 1: Denied boarding estimation framework using AFC and AVL data.

Mixture models are often unstable. To improve the stability of the estimation, we incorporate additional information. The first component mean $\mu_1$ is the expected journey time for passengers boarding the first train:

$$\mu_i = E\left[ t^a + \left( t^w + t^e + t^t \right) \right] = \beta_1 \times X + C_1$$

(2)

$t^a$ is the access time from the tap-in fare gate to the platform; $t^w$ the waiting time until the arrival of the first train; $t^e$ the extra waiting time due to denied boarding; $t^t$ the in-vehicle time; and $t^e$ the egress time from the platform to the tap-out fare gate after alighting; $X$ is a set of explanatory variables, such as average headway $\bar{h}$; $\beta_1$ and $C_1$ are parameters to be estimated. The parameters capture effects such as the impact of headway irregularity on waiting time for the first train, and different access/egress and in-vehicle times (since although all these passengers board the first train, they do not necessarily board the same train).

Furthermore, passengers who belong to group $k + 1$ experience average journey times approximately one headway longer than the average journey time of those in group $k$:

$$\mu_{k+1} - \mu_k = \beta_{k+1} \times \bar{h} + C_{k+1}$$

(3)

where $\beta_{k+1}$ and $C_{k+1}$ are parameters to be estimated. They capture effects similar to the above considerations (headway variability, lower walk speeds in group $k+1$ compared to group $k$, etc.). The cluster parameters are estimated by maximizing the log-likelihood of observing the journey times in the sample, given the above relationships between the parameters.

$$\text{Maximize } \sum_{t \in \Omega} \log\left(p\left(t_i; \Omega, \Sigma\right)\right)$$

s.t. \sum_{k} \omega_k = 1

$$\omega_k \geq 0, \forall k \in K$$

$$\mu_i = \beta_1 \times \bar{h} + C_1$$

$$\mu_{k+1} - \mu_k = \beta_{k+1} \times \bar{h} + C_{k+1}, \forall k \in K - 1$$

(4)

Constraints in mixture models have mainly been introduced to deal with the unboundedness of the likelihood function. A generalized Expectation and Maximization (EM) algorithm is developed for the solution of this problem on basis of the study in (Chauveau and Hunter, 2013).
B) Capacitated Assignment
We also propose an event-based assignment model, which takes as input the OD flows, path choice fractions, train movement data, and train capacity data, and outputs train load, crowding on platforms, passengers denied boarding, wait times, etc. for a given day.

Figure 2 illustrates the structure of the model. An event is the train arrival at, or departure from a station ordered chronologically. New and transferring passengers join the waiting queue on the corresponding platform and board a train on a FCFS basis, depending on available train capacity. The model uses AVL data for train arrival and departure times at stations. Other inputs include OD flows by time period, route choice fractions, and train capacity.

![Figure 2: Event-based assignment with capacity constraints and priority](image)

Case Study
A case study will focus on the application of the proposed and existing methods using data from the MTR network, for which denied boarding observations from manual surveys are also available. Figure 3 shows the evolution of the journey time distribution for passengers traveling during the evening peak period (18:00-19:00) from a busy station to a given destination.

![Figure 3: Journey time distribution between 17:30-19:30 in 2 minute intervals](image)

Most passengers have journey time around 6.5 minutes during the shoulder periods (17:30-18:00 and 19:00-19:30), and experience longer travel times, from 8 to 16 minutes, during the period (18:00-19:00), mostly impacted by the denied boarding at the origin station.
The proposed mixture model was applied using a sample of the above data for the evening peak period 18:00-19:00 on the same day as the manual survey. Figure 4 compares the estimation results against the survey, and the results from the application of the model in (Zhu et al., 2017). The mixture model results are similar to both the survey data and Zhu’s model, although it uses less information (no walk speed distribution is required).

The paper will include a comprehensive evaluation of the models for their accuracy and robustness using actual and synthetic data.

References
Chauveau, D., Hunter, D., R., 2013. ECM and MM algorithms for normal mixtures with constrained parameters. preprint http://hal.archives-ouvertes.fr/hal-00625285.


