Train dispatcher desk districting problem in high-speed railway network

Jun Zhao · Dian Wang · Qiyuan Peng

Abstract This paper considers the train dispatcher desk districting problem with a fixed number of desks for distributed dispatching and commanding of high-speed railway network from a strategic perspective. The problem aims to partition the stations and sections in the network into disjoint districts, and assign the districts to different train dispatcher desks. By considering many practical requirements such as districting feasibility, district contiguous and workload balance, we formulate the problem as a mixed integer linear programming model to minimize the weighted sum of total workload deviation of dispatcher desks and total coordination workload between dispatcher desks. An improved network flow-based technique is used to analytically represent the complicated district contiguous requirement using a polynomial number of constraints. Three families of valid inequalities are proposed to strengthen the model by exploiting the characteristics of the model. An iterative search algorithm embedded with intensification and neighborhood search procedures is developed to solve large-scale problem instances. Finally, realistic cases based on the high-speed railway network of Guangzhou Railway Corporation in China are constructed to test the proposed approaches.

Keywords High-speed railway · Districting · Train dispatcher desk

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1 Introduction

A high-speed railway network is usually distributed into different bureaus according to its precincts to efficiently and safely implement daily dispatching and commanding operations. Meanwhile, each bureau further partitions its network into several districts and assigns them to corresponding specific dispatcher desks for professional management. Train dispatcher desk is one of the most important desks in high-speed railway dispatching commanding system. It is not only the key commanding agency to guarantee safe and on-time operations of high-speed trains, but also its districting scheme is the basis to set other dispatcher desks. Therefore, it is of great theoretical and practical significance to study the train dispatcher desk districting problem in high-speed railway network. A good districting scheme could help to enhance the dispatching and commanding efficiency and to optimize the scheduling resource allocation effectiveness of high-speed railway network.

The train dispatcher desk districting problem in high-speed railway network aims to completely partition the stations and sections in the network into several districts administrated by corresponding train dispatcher desks, so that several operational and management requirements are satisfied. This problem is a typical districting application with extra specific constraints. The classical districting problem has widely application fields and was described in detail by Kalcsics (2015). Although different application fields have personalized districting criterions, the common criterions include workload balance, district contiguous, and district compactness, etc. Workload balance is that each district has equal or nearly equal amount of support votes, sales volume or workload. District contiguous guarantees that each district is a connected graph, i.e. every two geographic units in the same district are accessible in the district. District compactness requires that from geometry districts should present a square or circle round-shaped structure and avoid odd-shaped structures such as octopus or eel-like one.

As far as we know this problem has not been formally studied before by operations research approaches. A lot of researches focusing on the districting problems arising from other application fields have been done during the past 30 years. Typical applications can mainly be divided into four classes including political districting (Bozkaya et al, 2011; Ricca et al, 2013; Gopalan et al, 2013), sales territory districting (Kalcsics et al, 2005; Ríos-Mercado and Fernández, 2009; Salazar-Aguilar et al, 2011), services districting (Caro et al, 2004; Perrier et al, 2008; Steiner et al, 2015; Camacho-Collados et al, 2015), and administrative areas districting in the fields of logistics and transportation (Jarrah and Bard, 2012; Etemadnia et al, 2014; García-Ayala et al, 2016; Xie and Ouyang, 2016). The transportation administrative areas districting has the closest relationship to the problem investigated in this paper. Etemadnia et al (2014) investigated the problem of network partitioning for distributed traffic management applications. They formulated their problem as a mixed integer linear programming models to minimize the inter-flow among resulting sub-networks while satisfying the requirements of district contiguous and
workload balance. Xie and Ouyang (2016) studied the railroad caller districting problem under the constraints of reliability, contiguity, balance and compactness of caller desks. A large-scale mixed integer linear programming model with minimum composite objective was established.

From above, the train dispatcher desk districting problem in high-speed railway network has not attracted extensive attentions till now. There are still several issues which urgently need to be addressed when applying the existing districting models to solve this problem. Firstly, in existing districting models basic geographic units are usually only expressed as nodes or links. However, we need to partition stations and sections in the high speed railway network simultaneously for the train dispatcher desk districting problem. Secondly, most existing works aim to design more balanced districting schemes and only a few works consider the coordination workload between districts as districting criterion. For the train dispatcher desk districting problem, to control the desk workload and keep the desk independence, we have to take into consideration the internal workloads of dispatcher desks and the coordination workload between dispatcher desks together. Finally, most researchers focus on developing practically efficient solution algorithms according to the specific characteristics of their districting problems. Few of them attempt to establish theoretically rigorous and well-established mathematical models. To the best of our knowledge, only a few studies consider and formulate the district contiguous requirement exactly with a polynomial number of constraints. Shortcomings such as convergence difficulties still exist to be overcome among the existing technologies.

This paper tries to optimize the train dispatcher desk districting problem in high speed railway network by using advanced integer programming and graph theory. A mixed integer linear programming model is established to minimize the weighted sum of total workload deviation of dispatcher desks and total coordination workload between dispatcher desks under the restriction of many practical requirements such as districting feasibility, district contiguous and workload balance. Existing network flow-based technique is improved to exactly represent the district contiguous requirement. Valid inequalities including symmetry breaking, redundant flow elimination and elementary cycle breaking are proposed to strength the model. An iterative search algorithm embedded with intensification search and neighborhood search procedures is designed to solve large scale problem instances. Realistic case based on the high speed railway network of Guangzhou Railway Corporation in China is constructed to test the proposed approaches. Meanwhile, effects of main parameters are explored as well to provide managerial insights for decision makers.

The rest of this paper is organized as follows. A detailed problem description is firstly presented, and then we formulate the problem as a mixed integer linear programming model with all objectives and constraints. Later several valid inequalities are proposed to enhance the model and an iterative search algorithm is developed. Next, a realistic case is constructed to test the proposed approach. The effects of main parameters are also explored in this section. Finally, we conclude the main research works in this paper.
2 Problem description

There are numerous applications of districting problems in practice, each of which has special districting criterions and standards. To simplify the following discussion, it is necessary to describe the train dispatcher desk districting problem. Given the physical information of high-speed railway network (topological structure, station length, section length, etc.), and train dispatching workload (section workload = number of trains travelling \( \times \) section length, station workload = number of trains stopped \( \times \) station length), the train dispatcher desk districting problem aims to partition stations and sections in the network completely into disjoint districts served by corresponding train dispatcher desks, so as to satisfy and optimize specific districting criterions and standards.

In order to obtain practically feasible train dispatcher desk districting schemes of high speed railway network, several specific criterions including districting feasibility, district contiguous, workload balance, etc. should be satisfied. Districting feasibility is designed to reflect the characteristics of the train dispatching and commanding in China. It requires that each station and section should be assigned to a certain dispatcher desk uniquely due to the principle of centralized and unified commanding adopted in practice. Meanwhile, two stations connected by one section can be assigned to different desks, but a section and at least one connected station should be assigned to the same desk. Districting feasibility is the essential requirement when setting train dispatcher desks and it should be strictly satisfied. District contiguous requires that each district should be connected, i.e. every two stations in the same district are accessible in the district so as to simplify the daily operations of train dispatchers and to avoid repeated exchange of trains. Although this constraint only applies to stations, the districting feasibility constraint can avoid the situation that a dispatcher desk only governs an isolated section and it can ensure that each station and section administrated by a dispatcher desk are accessible together with the district contiguous requirement. Workload balance is to make an average allocation of the workload among all dispatcher desks such that the workload of each desk is in an allowable deviation range specified by the average workload. This constraint is soft and can be violated due to the difficulty of absolute balance, while the total deviation is controlled by the objective function described below.

Specific objective should be minimized when partitioning the high speed railway network to specific train dispatcher desks so as to provide practical feasible districting schemes according to the train dispatching and commanding characteristics in China. On one hand, the absolute balance of workload among dispatcher desks are difficult to be achieved in practice, hence it is necessary to minimize the total workload deviation of dispatcher desks, so as to avoid operation delay and work error of dispatchers caused by excessive workload, and vigilance decline of dispatchers and waste of dispatching resources due to exiguous workload. Together with the workload balance constraint, the total workload deviation objective can guarantee that the workload of each desk is
controlled in a reasonable range as much as possible. On the other hand, the total coordination workload between dispatcher desks should also be minimized. When the two stations connected by one section are assigned to different desks, then the dispatching and commanding operations of the corresponding trains should be accomplished together by these desks and coordination workload occurs. Hence, it is necessary to minimize the total coordination workload to maintain the relative independence of dispatcher desks. Hence, a minimum compound objective composed of the total workload deviation of dispatcher desks and the total coordination workload between dispatcher desks is designed in this paper to reflect the feature of the original problem.

This paper concentrates on developing theoretical rigorous and practical effective solutions for the train dispatcher desk districting problem in high-speed railway network from a macro perspective and ignoring the concrete details of middle and micro perspectives. To simplify the establishment of the optimization model, the following assumptions are designed.

1. The dispatching commanding operations of high-speed railway and conventional railway are implemented separately in China. Hence the train dispatcher desk districting problem arising from the conventional railway network is not considered in this paper.
2. The high-speed railway network investigated is essentially contiguous. If not, the corresponding train dispatcher desk districting problem can be divided into several simpler similar sub-problems according to its connected subsections.
3. There are no scientific and systematical methods to measure the dispatching and commanding workload of train dispatchers now. The number of trains and length of sections are usually applied to expressed workloads when districting train dispatcher desks in China. In this paper, station workload is further taken into consideration. We let section workload = number of trains travelling × section length, and station workload = number of trains stopped × station length. Meanwhile, other influence issues to the workloads are omitted.
4. District compactness requirement is extensively adopted in existing researches on the districting problem. However, the high speed railway network is composed by main stems, branches and junctions which makes the network presenting a banding structure. Thus the district compactness requirement is incompatible with the high speed railway network. It is not discussed in this paper.

3 Methodology

3.1 Notation

The train dispatcher desk districting problem in high-speed railway network is formulated as a mixed integer linear programming model in this paper. For simplicity, in what follows, the train and network denote the high-speed
train and high-speed railway network, respectively. The dispatcher desk and
districting scheme stand for the train dispatcher desk in the high-speed railway
and the districting scheme of high-speed railway, respectively. The main sets,
indices, parameters and decision variables used in the model are explained in
Table 1.

Table 1: Definition of sets, indices and decision variables

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1(V, E)$</td>
<td>High-speed railway network, where $V$ and $E$ are the set of stations and sections, respectively.</td>
</tr>
<tr>
<td>$</td>
<td>V</td>
</tr>
<tr>
<td>$i, j$</td>
<td>Station index, $i = {1, 2, \ldots,</td>
</tr>
<tr>
<td>$(i, j)$</td>
<td>Section index, $(i, j) \in E$.</td>
</tr>
<tr>
<td>$K,</td>
<td>K</td>
</tr>
<tr>
<td>$k, k'$</td>
<td>Dispatcher desk index, $k = {1, 2, \ldots,</td>
</tr>
<tr>
<td>$w_{ij}$</td>
<td>Dispatching and commanding workload of section $(i, j) \in E$.</td>
</tr>
<tr>
<td>$s_i$</td>
<td>Dispatching and commanding workload of station $i \in V$, respectively.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Allowable relative deviation to obtain the reasonable range of dispatcher desk workload.</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>Average workload of dispatcher desks calculated by the total workload and the number of desks.</td>
</tr>
<tr>
<td>$\beta, \gamma$</td>
<td>Objective weights of total workload deviation of dispatcher desks and total coordination workload between dispatcher desks, respectively, $\beta + \gamma = 1$.</td>
</tr>
<tr>
<td>$x_{ik}$</td>
<td>0-1 variable, 1 if station $i \in V$ is assigned to the district administrated by dispatcher desk $k \in K$, 0 otherwise.</td>
</tr>
<tr>
<td>$y_{ijk}$</td>
<td>0-1 variable, 1 if section $(i, j) \in E$ is assigned to the district administrated by dispatcher desk $k \in K$, 0 otherwise.</td>
</tr>
<tr>
<td>$v_{ij}$</td>
<td>0-1 variable, 1 if station $i$ and $j$ connected by section $(i, j)$ are assigned to different desks, 0 otherwise.</td>
</tr>
<tr>
<td>$b_k$</td>
<td>Nonnegative continuous variable, represents workload of dispatcher desk $k$.</td>
</tr>
<tr>
<td>$e_k$</td>
<td>Nonnegative continuous variable, represents absolute workload deviation of dispatcher desk $k$.</td>
</tr>
</tbody>
</table>

3.2 Objective function

Based on the problem description, a weighted minimum composite objective
function that reflects the total workload deviation of dispatcher desks and
the total workload coordination between desks simultaneously is designed and
expressed as follows.

$$
\min \beta \cdot \sum_{k \in K} e_k + \gamma \cdot \sum_{(i, j) \in E} w_{ij} \cdot v_{ij} 
$$

(1)

The first part of the objective is the total workload deviation of dispatcher
desks and the second part expresses the total workload coordination between
desks, while the objective weights are decided by the decision makers based
on their preference and satisfy $\beta + \gamma = 1$. 
3.3 Constraints

3.3.1 Districting feasibility

The dispatching and commanding operations of high-speed trains in China need to obey the principles of hierarchical management and centralized-unified commanding. As a consequence, each station and section in the network must be assigned to a certain dispatcher desk uniquely.

\[ \sum_{k \in K} x_{ik} = 1 \quad \forall i \in V \] (2)

\[ \sum_{k \in K} y_{ijk} = 1 \quad \forall (i,j) \in E \] (3)

A district served by a dispatcher desk composed by only one section is not allowed in practice to guarantee the district contiguous requirement. Therefore, a section and at least one connected station should be assigned to the same desk. Meanwhile, when the two stations connected by one section are assigned to a dispatcher desk, the section should also be assigned to the desk.

\[ y_{ijk} \leq x_{ik} + x_{jk} \quad \forall (i,j) \in E, \forall k \in K \] (4)

\[ x_{ik} + x_{jk} - 1 \leq y_{ijk} \quad \forall (i,j) \in E, \forall k \in K \] (5)

To ensure the reasonability when calculating the total workload coordination, decision variable \( x_{ik} \) and \( v_{ij} \) should satisfy the following relationship constraint.

\[ x_{ik} + \sum_{k' \in K|k' \neq k} x_{jk'} - 1 \leq v_{ij} \quad \forall (i,j) \in E, \forall k \in K \] (6)

3.3.2 District contiguous

Each district should be contiguous to simplify the daily train dispatching and commanding operations. The network flow-based technique proposed by Shirabe (2009) is the typical quantitative approach to exactly represent the requirement of district contiguous with a polynomial number of constraints. The existing technique suppose that each district has a center which locates at a certain station administrated by the corresponding desk, and the center of a district receives 1 unit of flow from every other station allocated to the district. The existing network flow-based technique proposed by Shirabe (2009) ignores the diversity of flow distributions and makes the proposed model loose, such that the bounding effectiveness of the linear relaxation model is not well and the convergence rate is relatively slow. Therefore, we attempt to improve the existing network flow-based technique by introducing a dummy sink denoted as \( t \) and supposing that each station generates 1 unit virtual flow and must flow into the sink. If station \( i \) is assigned to desk \( k \), then the flow generated
at station $i$ must first flow through the center of desk $k$ and finally flow into the sink.

A simple network with 6 stations and 6 sections shown in Figure 1 is used to describe the characteristics and requirements of using the improved network flow-based technique for district contiguous constraint. We have 4 representative flow distributions corresponding to the same districting scheme where station 6 serves as the center. In each sub-figure, blue thick arcs represent that there are flows on the arcs and the number near the arcs are the flow volume, while black thin arcs to the contrary. As observed, all the 4 flow distributions indeed produce the same districting result but hold different features. Sub-figure (1a) is cycle free and presents a tree structure, where sink $t$ is the root node and other stations are different levels of leaf nodes. The flow generated at each station flows from the superior leaf nodes to the inferior leaf nodes and finally flow into the root node. However, cycles occur in other sub-figures and the parameter $a$ in the last two ones can be an arbitrary non-negative constant. All the flow distributions satisfy the existing network flow-based technique proposed by Shirabe (2009) which makes the proposed model loose.

![Network flow distributions](image)

**Fig. 1: Improved network flow-based technique for district contiguous**

We improve the network flow-based technique for representing the district contiguous constraint by constructing a tree-structure flow distribution so as to eliminate the diversity of flow distributions. Hence, auxiliary sets, indices, parameters and decision variables to express the improved network flow-based technology should be introduced as in Table 2.

To guarantee a tree-structure flow distribution in each district as in sub-figure (1a), the following requirements should be satisfied. Firstly, directed arcs $(i,j) \in A$ and $(j,i) \in A$ for each section $(i,j) \in E$ cannot have flows at
Table 2: Definition of auxiliary sets, indices and decision variables

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_2(V, A')$</td>
<td>Auxiliary directed graph, where $A'$ is a directed arc set and $A' = A \cup D$.</td>
</tr>
<tr>
<td>$A$</td>
<td>Directed arc set. For each section $(i, j) \in E$, two directed arc $(i, j)$ and $(j, i)$ are constructed such that $A = {(i, j) \cup (j, i)</td>
</tr>
<tr>
<td>$D$</td>
<td>Directed dummy arc set. For each station $i \in V$ and the dummy sink $t$, a directed arc $(i, t)$ is introduced such that $D = {(i, t)</td>
</tr>
<tr>
<td>$c_{ik}$</td>
<td>0-1 variable, 1 if station $i$ is the center of desk $k$, 0 otherwise.</td>
</tr>
<tr>
<td>$f_{ijk}$</td>
<td>Nonnegative continuous variable, represents the flow of directed arc $(i, j) \in A'$ which flow through the center of desk $k$ and flow into the sink.</td>
</tr>
<tr>
<td>$z_{ijk}$</td>
<td>0-1 variable, 1 if directed arc $(i, j) \in A'$ has flows which flow through the center of desk $k$ and flow into the sink, 0 otherwise.</td>
</tr>
</tbody>
</table>

the same time. Secondly, for each station, there must only be one arc having flows among all directed arcs starting from the station. Thirdly, for each station, there can be more than one arcs having flows among all directed arcs ending at the station. Finally, only the arcs starting from the centers can have flows flowing into the sink. Besides, if the sub-figures in Figure 1 represent four different districts and connected arcs between these districts are omitted, then the flow distribution between different districts has the following characteristics. Firstly, in each district, there cannot be any flow which flows through the center of other districts and then flows into the sink. Secondly, the arcs connecting different districts cannot have any flow.

Thus, the detailed modeling processes to represent the district contiguous constraint by the improved network flow-based technique is described as follows. Firstly, each district must have a unique center which should be administrated by the corresponding dispatcher desk as well.

$$\sum_{i \in V} c_{ik} = 1 \quad \forall k \in K$$ (7)

$$c_{ik} \leq x_{ik} \quad \forall i \in V, \forall k \in K$$ (8)

Then the requirement of flow conservation for each station and each district should be satisfied. Equation (9) represents that when station $i$ is assigned to desk $k$, then station $i$ will generate 1 unit flow which must firstly flow through the center of desk $k$ and finally flow into the sink, otherwise the flows flowing into and out station $i$ should be equal. Equation (10) ensure that the total flows flowing into the sink from each district equals to the total flows generated at the stations administrated by the corresponding desk.

$$\sum_{j\{i, j\} \in A'} f_{ijk} - \sum_{j\{j, i\} \in A'} f_{jik} = x_{ik} \quad \forall i \in V, \forall k \in K$$ (9)

$$\sum_{i\{i, i\} \in A'} f_{itk} = \sum_{i \in V} x_{ik} \quad \forall k \in K$$ (10)

Finally, to guarantee a tree-structure flow distribution in each district, the requirements, i.e. directed arc $(i, j) \in A$ and $(j, i) \in A$ cannot have flows at
the same time for each section \((i, j) \in E\) represented by equation (11), at most one arcs can have flows for all arcs starting from each station expressed by equation (12), and only the centers can have flows flowing into the sink described as equation (13), should be satisfied. Besides, the flow on each arc is normalized by equation (14).

\[
\begin{align*}
z_{ijk} + z_{jik} &\leq x_{ik} \quad \forall (i, j) \in A, \forall k \in K \quad (11) \\
\sum_{j \mid (i, j) \in A^0} z_{ijk} &\leq x_{ik} \quad \forall i \in V, \forall k \in K \quad (12) \\
z_{ikt} &\leq c_{ik} \quad \forall i \in V, \forall k \in K \quad (13) \\
f_{ijk} &\leq (|V| - |K| + 1) \cdot z_{ijk} \quad \forall (i, j) \in A^0, \forall k \in K \quad (14)
\end{align*}
\]

3.3.3 Workload balance

As the railway network represents a special eel-like structure, it is difficult to achieve a perfect workload balance for all dispatcher desks. Therefore, it is allowed that the actual workload of each desk can be outside the given reasonable range. However, the total workload deviation is controlled by the first part of the objective function.

\[
\begin{align*}
b_k &= \sum_{(i, j) \in E} w_{ij} \cdot y_{ijk} + \sum_{i \in V} s_i \cdot x_{ik} \quad \forall k \in K \quad (15) \\
e_k &\geq b_k - (1 + \alpha) \cdot \bar{w} \quad \forall k \in K \quad (16) \\
e_k &\geq (1 - \alpha) \cdot \bar{w} - b_k \quad \forall k \in K \quad (17)
\end{align*}
\]

Equation (15) is designed to calculate the actual workload of each dispatcher desk which equals to the total workload of administrated stations and sections of the desk, while the workload deviation of each dispatcher desk is expressed by equation (16) and (17) together.

3.4 Mathematical Model

Overall, the train dispatcher desk districting problem in the high speed railway network can be formulated as an optimization model \([M]\) to minimize objective (1) and satisfy constraints (2) to (17).

The problem is NP-hard as it can be reduced to a multiterminal cut problem proposed by Dahlhaus et al (1994). Given the undirected graph \(G = (V, E)\), the non-negative weight of each edge \(w_e (\forall e \in E)\) and a specific node set \(S = \{s_1, s_2, \ldots, s_k\} \subseteq V\), the multiterminal cut problem is to obtain a minimum cut set \(E' \subseteq E\) such that each node in \(S\) is not connected with others when removing \(E'\) from \(G\), i.e. to divide \(G\) into \(k\) disjoint districts and each district has only one node included by \(S\). For the original problem, assume that the district centers are given in advance and the district contiguous
and workload balance are not considered. Further, station workload and the total workload deviation of dispatcher desks are omitted. Then the high-speed railway network and the district centers can be considered as graph $G$ and node set $S$, respectively, and the section workload and the total coordination workload between dispatcher desks are the weight of edge $w_e$ and the cut set $E'$, respectively. Thus the original problem is reduced as a multiterminal cut problem. The latter problem has been proved to be NP-hard by Dahlhaus et al (1994) when $k \geq 3$. The desk number in the original problem is usually more than 2, thus the problem is NP-hard.

Meanwhile, model $[M]$ is a mixed integer linear programming model and can be quickly solved to optimality by state-of-the-art commercial optimization software for small scale problems. To solve large scale problems effectively, three families of valid inequalities are designed based on the features of the optimization model.

4 Valid inequalities

4.1 Symmetry breaking

The flow distribution in each district may cause internal symmetry since each station in the same district can be the corresponding center which causes different flow distribution but the same districting scheme. The internal symmetry caused by the flow distribution is illustrated by sub-figure (2a) and (2b), in which station 6 and station 3 are the center of the district, respectively.

Fig. 2: Illustration of the valid inequalities
To break the internal symmetry, assume that station with the largest index among the stations administrated by a desk should be the corresponding center and it is expressed as equation (18). Hence, as shown by equation (19), if station $i$ is the center of desk $k$, then any station $j > i$ cannot be assigned to the desk at the same time.

$$i \cdot x_{ik} \leq \sum_{j \in V} j \cdot c_{jk} \quad \forall i \in V, \forall k \in K$$ (18)

$$\sum_{j \in V|j > i} x_{jk} \leq \min \{|V| - i, |V| - |K|\} \cdot (1 - c_{ik}) \quad \forall i \in V, \forall k \in K$$ (19)

Besides, there is no difference between dispatcher desks except the initial index. However, different permutations of desks may represent the same districting scheme, and hence incurs external symmetry. For example, the district shown in sub-figure (2a) and (2c) could be allocated to desk $k$ and $k'$, respectively, while the opposite is also feasible in practice. To break this symmetry, suppose that a dispatcher desk with a larger index will have a larger index of center.

$$\sum_{i \in V} i \cdot c_{ik} \leq \sum_{j \in V} j \cdot c_{jk'} - (k' - k) \quad \forall k, k' \in K |k < k'$$ (20)

### 4.2 Redundant flow elimination

As seen from sub-figure (2a) and (2c), for each district, there is no flow which flows through the centers of other desks and then flows into the sink. Meanwhile, if station $i$ and station $j$ are assigned to different desks, then directed arc $(i, j) \in A$ and $(j, i) \in A$ would not have any flow. For example, station 5 in sub-figure (2a) and station 9 in sub-figure (2c) are allocated to different districts, therefore, if these two stations are directly connected by directed arcs, then there will not be any flow on these arcs.

$$\sum_{k \in K} z_{ijk} + \sum_{k \in K} z_{jik} \leq 1 - v_{ij} \quad \forall (i, j) \in E$$ (21)

Besides, equation (12) and (13) guarantee that, for all directed arcs starting from the centers, only the arcs ending at the sink will have flows, i.e., there will be no flow flowing from the centers to other stations.

$$\sum_{j|(i, j) \in A} z_{ijk} \leq 1 - c_{ik} \quad \forall i \in V, \forall k \in K$$ (22)
4.3 Elementary cycle breaking

Beyond the internal and external symmetry mentioned above, another symmetry may occur when there are multi routes between two stations, leading to elementary cycles in the high speed railway network. For example, elementary cycle (2, 3, 6, 5, 4, 2) is contained in the network shown in Figure 2, thus the symmetry caused by this cycle could be expressed by sub-figure (2a) and (2d). Note that this symmetry occurs only when all sections in an elementary cycle are allocated to the same district.

To break the elementary cycle symmetry, this paper supposes that when all sections in an elementary cycle denoted as $E_r$ is allocated to a certain district, where $R$ and $r$ are the elementary cycle set and index in the high speed railway network, respectively, then for section $(i, j) \in E_r$ which has the lowest workload among $E_r$, the corresponding directed arc $(i, j) \in A$ and $(j, i) \in A$ would not have any flow. For example, there is not any flow on directed arc (2, 4) and (4, 2) in sub-figure (2a).

$$\sum_{k \in K} (z_{ijk} + z_{jik}) \leq \sum_{(i', j') \in E_r} v_{i'j'} \quad \forall s \in S, w_{ij} = \min\{w_{i'j'}|(i', j') \in E_r\} \quad (23)$$

There is usually not too many elementary cycles in high speed railway network, hence these elementary cycles can be obtained effectively in advance by the method described by Lee (1982).

5 Solution method

Preliminary computation results show that the valid inequalities can accelerate the solution speed for small scale problems, but convergence difficulties still exist for large scale problems. Besides, if centers are given in advance, then our models can solve large scale problems to optimality in short time. Therefore, an iterative search algorithm embedded with intensification and neighborhood search procedures is designed to solve large scale problems effectively.

The main procedures of the algorithm are as follows. Firstly the center of districts is initialized by solving a specific $p$-median model which is elaborated by Daskin and Maass (2015). Then the intensification search procedure is implemented based on the initial centers by solving a specific districting model which can be the original optimization model or together with several families of valid inequalities. Next, several connected districts are adjusted to improve the intensification districting scheme and the neighborhood solutions are constructed. The intensification and neighborhood search procedures are performed iteratively until the termination conditions are satisfied.

5.1 Initialization

Initialized centers are provided to the subsequent intensification search by solving a specific $p$-median problem studied by Daskin and Maass (2015).
Suppose that stations are regarded as customers and facilities simultaneously, and the demand of each customer equals to the sum of the corresponding station workload and half of the total section workload connected with the station. The unit cost between customer and facility equals to the shortest distance between the corresponding stations. Further, the section assignment, district contiguous and workload balance requirements are ignored. Meanwhile, the capacity of each facility cannot exceed the workload upper limitation, and the objective is only to minimize the total demand weighted cost. Thus the original problem is reduced as a capacitated p-median problem where p is the number of desks. If an optimal solution of the p-median problem is obtained, then the locations of facilities are chosen as the initial centers of districts denoted as \( I_K = \{ i_k | k \in K \} \), where \( i_k \) is the center of desk \( k \). The requirement that the desk with a smaller index has a smaller center index should be satisfied.

5.2 Intensification search

The intensification search procedure is conducted by fixing variable \( C = \{ c_{ik} | i \in V, k \in K \} \) with the current desk centers \( I_K \) and then solving the districting model to optimality to obtain a feasible districting scheme. The resulting districting scheme and objective value are denoted as \( IS_X \) and \( IS_OFV \), respectively. Note that initial desk centers obtained by the capacitated p-median problem can balance the workload at a certain extent, but the districting model may be infeasible when the current desk centers are the initial value and the internal symmetry breaking valid inequality are introduced to enhance the model, on this occasion, it is necessary to relax equation (18) and (19). After this intensification search, the center of each desk is adjusted to be the administrated station with the largest index, such that in following intensification search these two valid inequalities can be used to accelerate the solution speed.

5.3 Neighborhood search

The neighborhood search procedure is implemented by redistricting \( m \geq 2 \) connected districts \( K' = \{ k_1, k_2, \ldots, k_m \} \) based on \( IS_X \). For each section \((i,j) \in E\), if station i and j are administrated by desk \( k_1 \) and \( k_2 \), respectively, then desk \( k_1 \) and \( k_2 \) are adjacent; if desk \( k_1 \) and \( k_2 \) are adjacent while \( k_2 \) and \( k_3 \) are also adjacent, then desk \( k_1, k_2 \) and \( k_3 \) are connected, and so forth.

The procedures of neighborhood search is described as follows. Firstly, for each dispatcher desk \( k \in K' \setminus K' \), keeping the corresponding district unchanged, i.e. fixing variable \( x_{ik}, y_{ijk}, c_{ik}, f_{ijk}, z_{ijk}, b_k \) and \( e_k \). Then solve the districting model to find a better districting scheme. If an optimal solution is obtained or other termination conditions are reached, then save the obtained districting scheme and objective value of each neighborhood solution.
All combinations of \( m \) connected districts determined by the intensification search should be considered in the neighborhood search procedure. The best districting scheme \( NS_X \) and corresponding objective value \( NS_{OFV} \) are recorded after the neighborhood search procedure. Note that \( m \) is a user-defined parameter and should be determined by balancing the solution quality and computation time. A larger \( m \) may enhance the search effect but increase the solution difficulty of each neighborhood search, and a smaller \( m \) is just the opposite.

5.4 Termination conditions

In each iterative search, if the best neighborhood solution is better than the current intensification solution, i.e. \( (IS_{OFV} - NS_{OFV})/IS_{OFV} > \varepsilon \), where \( \varepsilon \) is the pre-determined allowable relative tolerance, and the total computation time does not reach the maximum allowable time, then update desk centers based on the best neighborhood solution, return to the intensification search procedure, and start the next iteration. Otherwise, output the districting scheme \( NS_X \) and corresponding objective value \( NS_{OFV} \), and algorithm is terminated.

6 Computational tests

6.1 Case setup

A realistic case based on the high speed railway network of Guangzhou Railway Corporation in China is used to test the proposed approaches. The operation length of the high speed railway is 2820.95 km until June 2016, and the numbers of stations and sections are both 116. The sketch map of the high speed railway network is shown in Figure 3.

According to technological documents of the train diagram used in 2016, the location, length and traffic volume of each segment of the test network are shown in Table 3. For simplicity, assume that the traffic volume of each section equals to that of its corresponding segment, and the number of trains handled at each station equals to the sum of traffic volume of connected sections. Besides, the detailed lengths of sections and stations are not given in this paper due to lack of paper.

The test high speed railway network is partitioned into 12 train dispatcher desks with the empirical method now as shown in Figure 3. Assume that the allowable relative workload deviation \( \alpha \) is 0.1, then the average workload of dispatcher desks is 45121 train-kilometers and the upper and lower limitation of the reasonable range are 49633 and 40609 train-kilometers, respectively. The statistic results of the current districting scheme are shown in Table 3. In this table, Column 2 and column 3 are the number of stations and total section length administrated by the desk, respectively. Column 4 represents the actual
Fig. 3: Sketch map of tested instance and current districting scheme

desk workload. Column 5 is the absolute workload deviation, where a negative and positive number indicates the actual workload is smaller than the lower limitation and larger than the upper limitation respectively, and zero means that the workload is in the reasonable range.

As seen from Figure 3 and Table 4, the difference of total section length administrated by dispatcher desks is relative large. The total section length of desk 3 is 448.942 km and that of desk 6 is only 80.955 km. Besides, the largest workload deviation among dispatcher desks reaches 85661 train-kilometers.
Table 3: Segment information of the test instance

<table>
<thead>
<tr>
<th>Index</th>
<th>Segment</th>
<th>Length (km)</th>
<th>Volume (pair)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-3</td>
<td>76.135</td>
<td>75</td>
</tr>
<tr>
<td>2</td>
<td>3-5</td>
<td>25.631</td>
<td>79</td>
</tr>
<tr>
<td>3</td>
<td>3-20</td>
<td>137.835</td>
<td>103</td>
</tr>
<tr>
<td>4</td>
<td>3-25</td>
<td>148.351</td>
<td>125.5</td>
</tr>
<tr>
<td>5</td>
<td>5-7</td>
<td>99.198</td>
<td>77</td>
</tr>
<tr>
<td>6</td>
<td>7-11</td>
<td>206.839</td>
<td>87</td>
</tr>
<tr>
<td>7</td>
<td>7-19</td>
<td>88.439</td>
<td>43</td>
</tr>
<tr>
<td>8</td>
<td>11-13</td>
<td>88.004</td>
<td>62</td>
</tr>
<tr>
<td>9</td>
<td>20-21</td>
<td>81.790</td>
<td>101</td>
</tr>
<tr>
<td>10</td>
<td>25-29</td>
<td>260.646</td>
<td>114.5</td>
</tr>
<tr>
<td>11</td>
<td>25-34</td>
<td>11.500</td>
<td>36</td>
</tr>
<tr>
<td>12</td>
<td>29-33</td>
<td>209.162</td>
<td>115.5</td>
</tr>
<tr>
<td>13</td>
<td>33-41</td>
<td>17.534</td>
<td>63.5</td>
</tr>
<tr>
<td>14</td>
<td>33-60</td>
<td>34.134</td>
<td>85.5</td>
</tr>
<tr>
<td>15</td>
<td>33-79</td>
<td>102.384</td>
<td>128</td>
</tr>
<tr>
<td>16</td>
<td>33-95</td>
<td>5.402</td>
<td>76</td>
</tr>
<tr>
<td>17</td>
<td>34-39</td>
<td>112.240</td>
<td>59</td>
</tr>
</tbody>
</table>

Table 4: Statistics results of current districting scheme

<table>
<thead>
<tr>
<th>Desk Index</th>
<th>Section Station</th>
<th>Length (train-km)</th>
<th>Workload (train-km)</th>
<th>Deviation (train-km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>206.150</td>
<td>32208</td>
<td>-8401</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>297.130</td>
<td>41138</td>
<td>73</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>448.942</td>
<td>96921</td>
<td>47288</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>423.838</td>
<td>99402</td>
<td>49770</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>137.020</td>
<td>17810</td>
<td>-22799</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>80.955</td>
<td>13742</td>
<td>-26867</td>
</tr>
</tbody>
</table>

Only the workload of desk 2 and desk 7 is in the reasonable range. The workload of desk 3, 4 and 11 is larger than the upper limitation in which desk 4 has the largest workload. The workload of other desks is smaller than the lower limitation and the workload of desk 6 is the smallest. Further assume that the two objective weights are both 0.5, then the objective value of the current districting scheme is 125718 train-kilometers, and the total workload deviation of dispatcher desks and total coordination workload between dispatcher desks are 209969 and 41467 train-kilometers, respectively.

In the following, the models and algorithm are all coded in MATLAB R2012b and CPLEX 12.5 is invoked to solve the models. The computations are executed on a PC with Inter Core i5-2400 3.1 GHz CPU, 8 GB RAM, and Windows 7-64 bits operation system.

6.2 Computational results

According to approach configurations, the proposed iterative search algorithm (ISA) with \( m \) equal to 6 and all three families of valid inequalities is adopted to solve the test case. The number of dispatcher desks and the value of \( \alpha \) are 12 and 0.1, respectively, and \( \beta \) and \( \gamma \) both equal to 0.5 as well for the
convenience of comparison with the empirical method. Our algorithm is terminated after 2 intensification searches and 32 neighborhood searches with the total computation time of 471 s. The objective value is 46343 train-kilometers and the total workload deviation of dispatcher desks and the total coordination workload between dispatcher desks is 6436 and 86251 train-kilometers, respectively. The optimized districting scheme is depicted in Figure 4 and the corresponding statistics results are summarized in Table 5.

Fig. 4: Optimized districting scheme obtained by the algorithm
As observed from Table 5, the difference in the total section length administered by dispatcher desks is not significant. The largest difference of total section length is just 130.885 km. Besides, the workload of desks is relatively balanced. All desks have a reasonable workload except for desk 1, 2, 4 and 11, in which the workload of the first three is smaller than the lower limitation and that of the last one is larger than the upper limitation.

By comparing the optimized districting scheme with the current one, we can conclude that in comparison to the current empirical method, our proposed iterative search algorithm holds the following features. Firstly, the objective value is decreased by 63.1%, hence the optimized scheme is better than the empirical scheme from the aspect of design objective. Secondly, the total workload deviation of dispatcher desks is dropped significantly. The number of desks whose workload is in the reasonable range is increased from 2 to 8. Further, even though the total coordination workload between desks is nearly doubled, its increment is far less than the decrement of the total workload deviation of desks, such that the objective value is still decreased significantly. Note that for simplicity the objective weights equal to 0.5 in the tested instance, by modifying the value of objective weights based on the preference of decision makers in practice, the total coordination workload can be less than the empirical scheme as well. Finally, the total computation time is less than 8 minutes for the proposed algorithm, which is obviously shorter than the time needed by the empirical method. Thus, our algorithm can obtain a much more balanced districting scheme than the empirical method in a quite attractive time, and hence it can be used in practice.

6.3 Sensitivity analysis

6.3.1 Effect of allowable relative deviation

There are two groups of important parameters in the train dispatcher desk districting problem. The first one is the allowable relative deviation $\alpha$ and the second one includes the objective weights, i.e. $\beta$ and $\gamma$. We firstly explore the effect of $\alpha$ to the districting scheme and computation time. Assume that the number of dispatcher desks is 12 and $\beta$ and $\gamma$ both equal to 0.5. Algorithm ISA is adopted to implement the sensitivity analysis and the maximum allowable computation time is 1 h. Parameter $\alpha$ is ranged from 0 to 0.5 with an increment of 0.05. The sensitivity analysis results are provided in Figure 5.
As seen from Figure 5, with the increase of $\alpha$, the constraints of train dispatcher desk districting problem are more relaxed, such that the objective value is monotonically decreasing. The total workload deviation of dispatcher desks is generally decreased with a relatively large fluctuation, while the total coordination workload between dispatcher desks is generally decreased as well but with a small fluctuation. Meanwhile, the maximum and minimum workload presents an increasing and decreasing tendency respectively, and has a fluctuation at a certain and a small degree respectively. Besides, increasing the value of $\alpha$ will expand the solution domain of the train dispatcher desk districting problem and hence has a remarkable influence to the computation time. When the value of $\alpha$ does not exceed 0.35, the computation time maintains less than 10 minutes. When $\alpha$ is larger than or equal to 0.4, the computation time uprushes to more than 20 minutes quickly. In brief, $\alpha$ has a relative large influence to the districting scheme. In order to obtain a well balanced districting scheme, the value of $\alpha$ should be relatively small.

6.3.2 Effect of objective weights

We now analyze the effect of objective weights to the computational results. As $\beta + \gamma = 1$, we only analyze objective weight $\gamma$. There are still 12 dispatcher desks and $\alpha$ equals to 0.1. Algorithm ISA is implemented and the maximum computation time is 1 h. The value of $\gamma$ is increased from 0 to 1 with an increment of 0.1. The sensitivity analysis results are presented in Figure 6.

As shown in Figure 6, with the increase of $\gamma$, the importance of the total workload deviation is decreased gradually and its value is increased monotonically. On the contrary, the total coordination workload is more important with a larger $\gamma$ such that it is monotonically decreased. Meanwhile, the objective value is increased slowly at the first 9 values of $\gamma$, and is decreased sharply at the last two values. When $\gamma$ is less than or equals to 0.5, i.e. the total workload deviation is more important, the difference of workload between dispatcher desks is relatively small and the total coordination workload is relatively large. When $\gamma$ is between 0.1 and 0.5, the same districting scheme is obtained.
only differing in the objective value. When $\gamma$ is further increased, i.e. the total coordination workload becomes more important, the workload difference and the total workload deviation is increased significantly. Thus, objective weights have a large influence to the districting scheme. Decision makers should determine the value of them carefully based on practical situations. Besides, objective weights also have an obvious effect on the computation time. The computation time of the iterative search algorithm is monotonically increased with the increase of $\gamma$, especially when $\gamma$ is larger than 0.7. The reason can be explained by that initial desk centers are determined by the capacitated p-median model which can reduce the total workload deviation at a certain degree. Hence, a smaller $\gamma$ makes the total workload deviation more important and can provide higher quality initial desk centers, which decreases the update number of desk centers and the total iterative search time.

7 Conclusion

Train dispatcher desk districting is the basis of setting other specific dispatcher desks in high speed railway network. It is of great importance to increase dispatching and commanding efficiency and to optimize the dispatching resource allocation effectiveness. However, this problem has not received great attentions in the literature. This paper tries to develop theoretically rigorous and practically efficient solution approaches for the problem with modern integer programming and network optimization theory. Firstly, a mixed integer linear programming model to minimize the weighted sum of total workload deviation of dispatcher desks and total coordination workload between dispatcher desks is formulated by considering many practical requirements such as districting feasibility, district contiguous and workload balance. Secondly, three families of valid inequalities are proposed to strength the model. Computational results demonstrate that the valid inequalities can accelerate the solution speed of small scale problems, but they are not applicable for large scale problems. Thirdly, an iterative search algorithm embedded with intensi-
fication and neighborhood search procedures is developed to solve large scale problems. Computational results show that the algorithm can obtain better solutions in shorter time compared with CPLEX. The optimized districting scheme is much more balanced compared with that obtained by the empirical method. Finally, effects of the main parameters are also explored, which can provide decision supports for the train dispatcher desk districting in practice.

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