Passenger-oriented Railway Timetable Rescheduling in a Complete Blockage

Shuguang Zhan · S. C. Wong · Qiyuan Peng · S.M. Lo

Abstract Trains normally run as scheduled in a non-disrupted situation. However, due to the external and internal factors, trains may deviate from the original timetable during the daily operations. To this end, the involved dispatchers are required to reschedule disrupted trains efficiently to send disrupted passengers to their destinations as soon as possible. We focus on the train rescheduling in a seriously disrupted situation where a track segment is completely blocked for a relatively long period of time, e.g., 2 hours. In this disrupted situation, trains cannot pass the disrupted segment during the disruption. Passengers will be influenced by the disruption and can not travel as scheduled. We will simultaneously reschedule trains and the passenger routes from both operator’s and passenger’s perspectives. Train rescheduling and passenger route choice problem is formulated by an Integer Linear Programming model based on a space-time network. Finally, we test our models on the Chinese high speed railway to show the difference between the timetable obtained by minimizing the train deviation and that obtained by minimizing total general passenger travel cost.

Keywords High speed railway · Train rescheduling · Track blockage · Integer linear programming

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1 Introduction

Due to the advanced train control systems, trains run as scheduled (according to the original timetable) in a non-disrupted situation. However, some external and internal factors, e.g., adverse weather and malfunctioning of railway infrastructure, may cause trains to deviate from the original timetable in daily operations. Therefore, disrupted trains need to be rescheduled immediately after the disruption occurs. This research focuses on train rescheduling in a seriously disrupted situation where both tracks of a double-track segment are temporarily blocked. In such a disrupted situation, no train can pass the disrupted segment during the disruption.

Two main train rescheduling strategies are utilized to handle disrupted trains in a complete blockage. One is that disrupted trains are stopped at intermediate stations ahead of the disrupted segment and wait until the disruption is over, and then they can continue their journey. We call this strategy Disrupted Trains Waiting Strategy (DTWS). The other one is that disrupted trains are short-turned in the appropriate stations adjacent to the disrupted segment before they arrive at the disrupted segment. We call this strategy Disrupted Trains Short Turning Strategy (DTSTS). The former is used on some Chinese high speed railways and Japanese high speed railways, see e.g. Zhan et al (2015) and Hirai et al (2009) for instance. The later is applied on the railway systems in some European countries, such as the Netherlands, see Nielsen et al (2012), Louwerse and Huisman (2013), Veelenturf et al (2016) and Ghaemi et al (2016, 2017, 2018). This research focuses on the first train rescheduling strategy where disrupted trains have to stop properly at intermediate stations before the disrupted location to wait for the disruption.

As discussed in Zhan et al (2015), we mainly need to decide the arrival and departure time, the departure order of trains in each station, the stopping stations of trains to wait for the disruption, as well as whether some trains require to be canceled. Most research reschedule trains from the operator’s point of view. That is, their objective is to minimize the train deviation and the number of canceled trains. However, minimizing the train deviation cannot guarantee that the disposition timetable is convenience for the disrupted passengers. For example, if the number of passengers on train $t_1$ is 200, and that on train $t_2$ is 400, it is obvious that we should give priority to train $t_2$ to minimize the total passenger travel cost. To reduce the impact of disruptions on passengers is the main concern for train rescheduling in a disrupted situation. Unlike the previous research Zhan et al (2015) and Veelenturf et al (2016), we not only consider the train delay and train cancellation from the operator’s point of view but also take the passenger inconvenience into account. Based on a space-time network, an integer linear programming model is formulated to simultaneously reschedule trains and optimize passenger route choice from a macroscopic level. Our research can help the involved railway managers to balance the trade-off between the benefits of the railway company and railway passengers. We test our model on the Beijing-Shanghai high speed railway line in China.
The main contributions of this research is that we have extended the model in Zhan et al (2015) to include the passenger route choice. Therefore, the current model can reschedule trains and passenger route choice simultaneously.

The remainder of this paper is organized as follows. Section 2 gives the related literature review. In section 3, we briefly describe our problem. Model formulations for both train rescheduling and passenger route choice problem is given in section 4. Section 5 presents the computational results. Finally, we conclude this research and discuss some future research in Section 6.

2 Literature review

During railway operations a timetable is loaded in the railway dispatching center. Trains run according to this timetable in a non disrupted situation. However, disruptions inevitably occur in daily operations and these disruptions require trains to deviate from the original timetable. Rescheduling trains in a disrupted situation is quite important for dispatchers. A number of previous papers focus on train rescheduling problems, see in three recent surveys Corman and Meng (2015), Cacchiani et al (2014) and Fang et al (2015).

In the following, we will focus on the most relevant research about train rescheduling in a blocked situation. In a partial and complete blockage, disrupted trains cannot pass the disrupted segment as scheduled during the disruption. Nielsen et al (2012) rescheduled rolling stock in seriously disrupted situation on the Dutch railway. They have to decided the rolling stock connection between short turning trains due to the fact that disrupted trains cannot passed the disrupted segment. Louwerse and Huismann (2013) formulated the train rescheduling problem as an integer programming model. A disposition timetable is obtained based on a Dutch railway line. To handle the transition from the original timetable to a disposition timetable and vice versa, Veelenturf et al (2016) have extended the model of Louwerse and Huismann (2013). In addition, they have partly taken the rolling stock rescheduling into account. Dollevoet et al (2017) integrate the timetabling, rolling stock rescheduling and crew rescheduling. They applied an iterative approach to reschedule railway timetable, rolling stock and crew in a closed loop. In a disrupted situation where both/all the tracks in a segment are blocked (complete blockage), disrupted trains are short-turned at the stations before the disrupted location, see in Ghaemi et al (2016). They focused on how to short turn trains in multiple stations adjacent to the disrupted segment during the disruption. However, in their research, only trains in one direction is considered. Furthermore, they only reschedule trains during the disruption, and the transition phase is out of consideration. For the papers Nielsen et al (2012), Louwerse and Huismann (2013), Veelenturf et al (2016), Dollevoet et al (2017) and Ghaemi et al (2016, 2018), disrupted trains are short turned during the disruption due to the track blockage. This strategy is normally used on the railways where seat reservation system is not applied.
However, in some railway systems, a seat reservation system exists. Therefore, allowing passengers to change trains is more complicated. Disrupted trains are stopped and waited at intermediate stations for the disruption instead of short turning. Hirai et al (2009) have conducted research on how to stop disrupted trains at appropriate stations in a completely blocked situation, which is called train stop deployment planning. However, they have not taken the train rescheduling after the disruption into account. Zhan et al (2015) have taken the train rescheduling during and after the disruption into account. In their research how to stop disrupted trains during the disruption and how to reschedule trains after the disruption are solved by a MILP model.

Most of the previously mentioned papers focus on train rescheduling from the railway operator’s perspective. They only emphasize on train deviations, e.g., total train delay, weighted train delay and train cancelation. However, the impact of a disruption on passengers are not explicitly considered, although it is significantly important for railway transportation system. Only relatively few papers consider passenger choice. Cadarso et al (2013) consider the passenger demand change in a disrupted situation in a rapid transit network. They reschedule trains based on the forecast demand after the disruption occurs. However, they obtain the passenger demand before the rescheduled timetable, and passengers do not exactly know their possible choices. Therefore, the interaction between the passenger demand and the disposition timetable is lack. Kroon et al (2015) reschedule railway rolling stock considering the dynamic passenger demand by an iterative way. The rescheduled timetable is given as an input. They first reschedule the rolling stock based on the given timetable, and then assign passengers to trains. According to the passenger assignment, they reschedule rolling stock again to well meet the passenger demand and minimize the rolling stock cost. Veeleenturf et al (2017) extend the work in Kroon et al (2015) by taking the timetable rescheduling into account. Therefore, they iteratively solve the timetable rescheduling, rolling stock rescheduling and passenger assignment. To more exactly consider the passenger route choice in a disrupted situation, Binder et al (2017b) reschedule railway timetable and passenger route choice simultaneously by an ILP model. However, they conclude that the model is quite time consuming for a real-world case, and more efficient algorithms are necessary to be introduced when applying it to solve real-time problems.

3 Problem description

In a complete segment blocked situation, disrupted trains cannot pass the disrupted segment during the disruption. Therefore, a large number of trains will deviate from their original timetable due to the disruption, especially on a long and busy railway line. In such a disrupted situation, the involved dispatchers have to reschedule disrupted trains efficiently based on the current capacity of the railway infrastructure and traffic status. Specifically, they have to decide the proper departure and arrival time and orders of trains, as well
as the probability of canceling some trains. In what follows, we will illustrate the train rescheduling problems in detail by giving a small example shown in Figures 1.

A part of railway line consists of 6 stations ($S_1$ to $S_6$) and 5 segments between these stations. Three inbound (trains $G_1$, $D_3$ and $G_5$) and three outbound trains (trains $G_2$, $D_4$ and $G_6$) run on this part of railway line, where “G” trains are high speed trains and “D” trains are medium speed trains. A disruption occurs in the segment between stations $S_3$ and $S_4$ at time $h_1$, and it is estimated to end at time $h_2$. We assume that intermediate stations $S_2$, $S_3$, $S_4$ and $S_5$ all have 2 tracks. Due to the disruption, the dispatchers have to stop these 6 trains in appropriate intermediate stations to wait for the disruption (see Figure 1). As soon as the disruption ends, they have to decide the proper departure orders and time of these trains. Furthermore, trains $D_3$, $G_5$ and $G_6$ depart from their origin station after the occurrence of the disruption. Therefore, the involved dispatchers can decide whether some of them require to be canceled according to the current information.

Fig. 1  Train rescheduling in a complete blockage

In addition, due to the disruption, passengers may not be able to take the scheduled train, especially for those who have booked the tickets of canceled trains. Therefore, we have to allocate some of the passengers to other trains to continue to send them to their destinations. Due to the cancelation of trains and the capacity limitation of a train, some passengers possibly cannot board any train. Thus, they have to leave the railway system. We give an artificial path to these rejected passengers, and a relatively high penalty, e.g., the time horizon $T$, is added if an artificial path is utilized. For passengers convenience, the passenger delay and number of passengers who cannot board any train should be minimized.

In this research, we want to minimize the total general passenger travel cost. For each passenger from its origin $o$ to its destination $d$, there are a set of paths. A path is a sequence of boarding, driving, waiting, transfer and alighting movements. We introduce a utility function with each alternative path, and we assume that passengers select the path with the highest utility if possible. The utility function of each path $p$ depends on the following attributes, refers to Robenek et al (2016). Due to the fact that it is unusual for passengers to
depart from their origin earlier than the planned time, the earlier departure is out of consideration in this research.

- **In-Vehicle Time** ($VT_p$): time spent by passengers in one or several trains along their path $p$, in minutes.
- **Waiting Time** ($WT_p$): time spent by passengers waiting between two consecutive trains in a station along their path $p$. The time that passengers waits for the first train at their origin is not included since this time is the late departure time.
- **Number of Transfers** ($NT_p$): number of times that passengers need to change trains along path $p$.
- **Late Departure** ($LD_p = \max(0, t - t_o)$): time difference between the actual departure time $t$ and the desired departure time of passengers from their origin $o$, in minutes.

In a disrupted situation, disrupted passengers are not necessarily needed to pay extra fee if they change trains. Thus we assume that the price of a trip is equal among all the paths from the same origin-destination pair. It is unnecessary to consider fee in the utility function. For passengers that choose path $p$ to go from their origin to their destination, the utility function is as follows:

$$U_p = -(VT_p + \beta_1 \times WT_p + \beta_2 \times NT_p + \beta_3 \times LD_p)$$ (1)

In equation (1), $\beta_1$ to $\beta_3$ are the weights for each type of time component described above. We change all the other types of times into in-vehicle time by parameter $\beta$.

4 Model formulation

4.1 Assumptions and Notations

To formulate the passenger-oriented train rescheduling problem, the basic assumptions are given as follows:

- Each station track is connected with the inbound and outbound main lines.
- Trains that have already entered the disrupted segment at the occurrence of the disruption can continue their journey during the disruption.
- Inbound (outbound) trains operate on the inbound (outbound) track in the segment.
- Passenger OD do not change after the disruption.

The basic notations that will be used in our model formulation are introduced, see in Table 1.

4.2 Space-time graph

To formulate our problem, we introduce a space-time graph $G(N, A)$, inspired from Nguyen et al (2001) and Binder et al (2017a,b). We first discrete time into
Table 1 Notations used in the model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Set of time, $t, t', t'', t''' \in T$</td>
</tr>
<tr>
<td>$N$</td>
<td>Set of physical nodes, e.g., a station, $i, j \in N$</td>
</tr>
<tr>
<td>$L$</td>
<td>Set of physical links connecting two neighbor physical nodes, $(i, j) \in L$</td>
</tr>
<tr>
<td>$K$</td>
<td>Set of train services, $k, k' \in K$</td>
</tr>
<tr>
<td>$L_k$</td>
<td>Set of links used by train $k$</td>
</tr>
<tr>
<td>$N$</td>
<td>Set of time-expanded nodes, $(i, t) \in N$</td>
</tr>
<tr>
<td>$N_{tr}$</td>
<td>Set of time-expanded train nodes, $(i, t) \in N_{tr}$</td>
</tr>
<tr>
<td>$N_{pa}$</td>
<td>Set of time-expanded passenger nodes, $(i, t), (j, t') \in N_{pa}$</td>
</tr>
<tr>
<td>$A_{tr}$</td>
<td>Set of time-expanded train arcs, $(i, t; j, t') \in A_{tr}$</td>
</tr>
<tr>
<td>$A_{pa}$</td>
<td>Set of time-expanded passenger arcs, $(i, t; j, t') \in A_{pa}$</td>
</tr>
<tr>
<td>$A^p_{tr}$</td>
<td>Set of available arcs for train $k$</td>
</tr>
<tr>
<td>$A^p_{pa}$</td>
<td>Set of available arcs for passenger group $p$</td>
</tr>
<tr>
<td>$\Omega(i, t; j, t')$</td>
<td>Set of train arcs conflicting with arc $(i, t; j, t')$</td>
</tr>
<tr>
<td>$\hat{T}$</td>
<td>The end time of the planned time horizon</td>
</tr>
<tr>
<td>$c_{p}(i, t; j, t')$</td>
<td>Passenger travel cost on passenger arc $(i, t; j, t')$ for passenger group $p$</td>
</tr>
<tr>
<td>$P$</td>
<td>The passenger group set, $p \in P$</td>
</tr>
<tr>
<td>$P^p_k$</td>
<td>The passenger group set including passengers booked train $k$, $p \in P^p_k$</td>
</tr>
<tr>
<td>$D_p$</td>
<td>The number of passengers for passenger group $p$</td>
</tr>
<tr>
<td>$o_k$</td>
<td>The origin of train $k$</td>
</tr>
<tr>
<td>$d_k$</td>
<td>The destination of train $k$</td>
</tr>
<tr>
<td>$o'_p$</td>
<td>The origin of passenger group $p$</td>
</tr>
<tr>
<td>$d'_p$</td>
<td>The destination of passenger group $p$</td>
</tr>
<tr>
<td>$e_k$</td>
<td>The earliest departure time of train $k$ from its origin</td>
</tr>
<tr>
<td>$l_k$</td>
<td>The latest arrival time of train $k$ at its destination</td>
</tr>
<tr>
<td>$e'_p$</td>
<td>The earliest departure time of passenger group $p$ from its origin</td>
</tr>
<tr>
<td>$l'_p$</td>
<td>The latest arrival time of passenger group $p$ at its destination</td>
</tr>
<tr>
<td>$q_k$</td>
<td>The capacity of train $k$</td>
</tr>
<tr>
<td>$C_{cap}$</td>
<td>The capacity of station $i$, also the total number of tracks in $i$</td>
</tr>
<tr>
<td>$de_{Ph, o_k}$</td>
<td>The planned departure time of train $k$ from its origin station</td>
</tr>
<tr>
<td>$T$</td>
<td>The maximum stop time of trains due to the disruption</td>
</tr>
<tr>
<td>$H_{start}$</td>
<td>The occurrence time of a disruption</td>
</tr>
<tr>
<td>$H_{end}$</td>
<td>The end time of a disruption</td>
</tr>
</tbody>
</table>

Time intervals, for example, $T = \{0, \tau, 2\tau, ..., n\tau\}$, where $T$ is the considered planning horizon. The length of time interval $\tau$ should be properly selected, which can be e.g., 1 minute or 30 seconds. In the space-time graph, a physical node $i$ such as a station in the railway system is expanded as node $(i, t)$ in the time dimension. Thus a physical link $(i, j)$ between two station nodes $i$ and $j$ is expanded as an arc $(i, t; j, t')$, which connects node $(i, t)$ and its following node $(j, t')$. This arc means that a train $k$ starts from node $i$ at time $t$ and arrives at node $j$ at time $t'$. For each train $k$, we assume that it starts from its dummy origin node $o_k$ and ends at the dummy destination node $d_k$, and its earliest start time from the origin and latest arrival time at its destination are $e_k$ and $l_k$, respectively. Since trains cannot depart from their origin before the scheduled time, the earliest departure time $e_k$ is assumed to be the planned departure time of train $k$. In addition, to ensure that all trains arrive at their destination, we set the latest arrival time as the end time of the time horizon $\hat{T}$. For each train
there are many passengers that have booked the tickets for it. We call all the passengers that have booked the tickets of the same train from the same origin station to the same destination station as a passenger group. Therefore, each passenger group has the same expected departure time, the scheduled departure time of the booked train. Due to the limited capacity of trains in a disrupted situation, we allow split passengers within a passenger group, and we assume the passengers within a group are homogeneous. For each passenger group, we assume a dummy origin \( o_p' \) and a dummy destination node \( d_p' \) for it. And the earliest start and latest arrival time are \( e_p' \) and \( l_p' \) respectively. Similarly, the earliest departure time \( e_p' \) is the planned departure time of the train that passenger group \( p \) has booked the ticket. The latest arrival time \( l_p' \) is the end of the time horizon \( T \). Based on the space-time network, the following types of arcs are defined, representing all feasible movements of trains and passengers:

- **Driving arcs** representing a movement of a train or a passenger group from one station to the following station. A driving arc connects a departure event at a station node \((i,t)\) to an arrival event of the following station node \((j,t')\). They are expressed by the set \( A_{dr} = \{(i,t);j,t')|(i,t),(j,t') \in N : i \neq j, \forall t,t' \in T : t' - t = TD(i,j)\};

- **Waiting arcs** representing trains or passengers within the trains wait at stations for other passengers to board and alight. A waiting arc connects an arrival event of station \((i,t)\) to a departure event of the same station \((i,t')\), which can be expressed as: \( A_{wa} = \{(i,t);i,t')|(i,t),(i,t') \in N, \forall t,t' \in T : t' - t = TW(i,i)\). To reduce the number of waiting arcs, we assume a unit waiting arc \((i, t; i, t + 1)\) with a waiting time of 1 minutes. If a train waits at a station for several minutes, we assume that it will use several unit waiting arcs in the model.

- **Access arcs** representing a train starts from its origin (e.g., shunting yard) to a station in the transportation system. This arc set is denoted as: \( A_{acc} = \{(o_k,t);i,t')|(o_k, t),(i, t') \in N : o_k \neq i, \forall t,t' \in T : t' - t = TP(o_k,i)\};

- **Egress arcs** representing a train final arrives at its destination, from a station of the transportation system to a shunting yard. This set can be expressed as: \( A_{egr} = \{(i,t);d_k,t')|(i, t),(d_k, t') \in N : i \neq d_k, \forall t,t' \in T : t' - t = TP(i,d_k)\};

- **Boarding arcs** representing passengers board a train from its origin. This arc set is denoted as: \( A_{boa} = \{(o_p', t);i,t')|(o_p', t),(i, t') \in N : o_p' \neq i, \forall t,t' \in T : t' - t = TB(o_p',i)\};

- **Leaving arcs** representing passengers final arrive at their destination and alight the train. This set can be expressed as: \( A_{lea} = \{(i,t);d_p',t')|(i, t),(d_p', t') \in N : i \neq d_p', \forall t,t' \in T : t' - t = TL(i,d_p')\};

- **Transfer arcs** representing passengers transfer from one train to another at a station in their middle journey. This set can be expressed as: \( A_{tra} = \{(i,t);i,t')|(i, t),(i, t') \in N, \forall t,t' \in T : t' - t = TR(i,i)\};

- **Penalty arcs** representing passengers that cannot travel from their origin to their destination by train due to the capacity limitation. This set can be
expressed as: $A_{pen} = \{(o', t; d'_p, t')|(o'_p, t), (d'_p, t') \in N, \forall t, t' \in T : t' - t = T\}$. In a penalty arc, a virtual train is assumed to run from the origin of passenger group $p$ to its destination.

All the above arcs can be divided into two types, train related arc set $A_{tr}$ and passenger related arc set $A_{pa}$. Train arc set $A_{tr} = A_{dri} \cup A_{wai} \cup A_{acc} \cup A_{egr}$, and passenger arc set $A_{pa} = A_{dri} \cup A_{wai} \cup A_{boa} \cup A_{lea} \cup A_{tra} \cup A_{pen}$. Note that a driving arc and a waiting arc are for both trains and passengers. In the above arcs, $TD(i, j)$ and $TW(i, i)$ are the running time and dwell time for a train in a segment and station, and they are also can be regarded as the running and waiting time for passengers. $TP(o_k, i)$ and $TP(i, d_k)$ are the train preparation times in its origin and destination stations. $TB(o'_p, i)$ and $TL(i, d'_p)$ can be regarded as passenger boarding and alighting time in the origin and destination. The cost for passengers use an arc $(i, t; j, t') \in A_{pa}$ is related to the arc type, which is given in Table 2.

<table>
<thead>
<tr>
<th>Arc name</th>
<th>Start node</th>
<th>End node</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boarding arc</td>
<td>$(o'_p, e'_p)$</td>
<td>$(i, t)$</td>
<td>$\beta_3 \times \max(0, (t - e'_p))$</td>
</tr>
<tr>
<td>Driving arc</td>
<td>$(i, t)$</td>
<td>$(j, t')$</td>
<td>$t' - t$</td>
</tr>
<tr>
<td>Waiting arc</td>
<td>$(i, t)$</td>
<td>$(i, t')$</td>
<td>$\beta_1(t' - t)$</td>
</tr>
<tr>
<td>Transfer arc</td>
<td>$(i, t)$</td>
<td>$(i, t')$</td>
<td>$\beta_2 + (t' - t)$</td>
</tr>
<tr>
<td>Leaving arc</td>
<td>$(i, t)$</td>
<td>$(d'_p, l'_p)$</td>
<td>$0$</td>
</tr>
<tr>
<td>Penalty arc</td>
<td>$(o'_p, e'_p)$</td>
<td>$(d'_p, l'_p)$</td>
<td>$T$</td>
</tr>
</tbody>
</table>

Due to the fact that trains may stop at intermediate stations to wait for the disruption for quite a long time, the number of possible waiting arcs for a train is tend to be large. Therefore, we will construct waiting arcs with a duration of the time unit, e.g., 1 minute. In case a train waits at a station for several minutes, it can be regarded as using several consecutive waiting arcs. However, the minimum dwell time of a train stops at a station for passengers boarding and alighting should be respected.

4.3 Basic space-time graph construction rules

In traditional train rescheduling problem, many operational rules should be respected, such as the minimum train running and dwell time constraints. Thanks to the space-time graph formulation, a lot of constraints can be included in the space-time network construction stage (Mahmoudi and Zhou (2016), Jiang et al (2017)).

(1) Train running and dwell time constraints

The time for a train running in a segment between two stations (nodes) $i$ and $j$, and dwelling in a station $i$ should be within a given time interval. These constraints can be inherently considered when construct the driving and waiting arcs. For a driving arc $(i, t; j, t') \in A_{dri}$, the duration time $TD(i, j)$ should
respect $TD(i, j)^{\text{min}} \leq TD(i, j) \leq TD(i, j)^{\text{max}}$. Parameters $TD(i, j)^{\text{min}}$ and $TD(i, j)^{\text{max}}$ are the minimum and maximum running times of a train in segment $(i, j)$. Similarly, the dwell time of each train in a station (node $i$) should be no less than the minimum dwell time $TW(i, i)^{\text{min}}$ and no larger than the maximum dwell time $TW(i, i)^{\text{max}}$, $TW(i, i)^{\text{min}} \leq TW(i, i) \leq TW(i, i)^{\text{max}}$.

(2) Train departure time constraints

In a disrupted situation, we assume that trains cannot depart from a station before the scheduled time. Therefore, passengers who have booked tickets for a train do not miss the train due to its earlier departure (Zhan et al (2015)). If a train $k \in K$ uses arc $(i, t; j, t') \in A_{\text{dri}}$ from a station (node $i$) to the following station (node $j$), the start time $t \geq t_k^i$, where parameter $t_k^i$ is the scheduled departure time of train $k$ from station $i$.

(3) No anticipation on the occurrence of the disruption

Before a disruption occurs, we assume that trains run as scheduled. That is, anticipation on the occurrence of the disruption is not allowed. Due to this assumption, trains have to use the same arc as scheduled before the disruption, which is helpful to reduce the number of possible used arcs.

(4) Prevent trains passing the disrupted segment

No trains can pass the disrupted segment during the disruption and they have to wait until the end of disruption. Therefore, if a train scheduled to pass the disrupted segment during the disruption, it has to enter the disrupted segment after the end of the disruption. For any disrupted train arc $(i, t; j, t')$, link $(i, j)$ is the disrupted link and $H_{\text{dis}}^{\text{start}} \leq t \leq H_{\text{dis}}^{\text{end}}$, no train $k \in K$ can use it.

4.4 ILP model formulation

Based on the graph defined in Section 4.2, we first introduce the following two decision binary variables corresponding to trains. The first one denotes whether train $k$ uses arc $(i, t; j, t')$, and the second one denotes whether train $k \in K$ needs to be canceled due to the disruption. Their definitions are as follows.

$$
x_k^i(t, t; j, t') = \begin{cases} 
1 & \text{if train } k \in K \text{ uses time-variant arc } (i, t; j, t') \\
0 & \text{otherwise}
\end{cases}
$$

$$
y_k = \begin{cases} 
1 & \text{if train } k \in K \text{ is canceled} \\
0 & \text{otherwise}
\end{cases}
$$

We also introduce an integer variable, which denotes the number of passengers in passenger group $p$ traveling on arc $(i, t; j, t') \in A_{\text{pa}}$. Note that for each train $k$, there are many different passenger groups $p \in P_k$ related to train $k$.

$$
v_p^i(t, t; j, t) : \text{the number of passengers in group } p \text{ uses arc } (i, t; j, t') \in A_{\text{pa}}
$$

To formulate the train rescheduling model, we first define an incompatible arc set $\Omega(i, t; j, t')$ for arc $(i, t; j, t')$ to model the departure and arrival headway
constraints between two trains, similar method see in Zhou et al (2017) for railway timetabling. For a segment track \((i, j)\), the incompatible arc set is defined as follows:

\[
\Omega_{(i,t,j,t')} = \{(i, t' ; j, t'') : |t - t'| \leq h^\text{min}_\text{dep} \cup |t' - t''| \leq h^\text{min}_\text{arr}\}
\]

(2)

where \(h^\text{min}_\text{dep}\) and \(h^\text{min}_\text{arr}\) are the minimum departure and arrival headway time between two trains in a station respectively.

In passenger’s perspective, they want to arrive at their destination as soon as possible. Recall that we assume that passengers choose routes according to the utility function of routes. We will minimize the total general travel cost for all the passengers.

\[
\min: Z = \sum_{p \in P} \sum_{(i,t,j,t') \in A^p_{pa}} c^p_{(i,t,j,t')} \times v^p_{(i,t,j,t')} \tag{3}
\]

subject to:

Train operational constraints:

\[
\sum_{(o_k,e_k,j,t') \in A^k_r} x^k_{(o_k,e_k,j,t')} = 1 - y_k \quad \forall k \in K
\]

(4)

\[
\sum_{(j',t',i) \in A^k_r: (i,t) \notin \{(o_k,e_k),(d_k,T)\}} x^k_{(j',t',i)} = \sum_{(i,t') \in A^k_r: (i,t) \notin \{(o_k,e_k),(d_k,T)\}} x^k_{(i,t',i)} \quad \forall k \in K
\]

(5)

\[
\sum_{(i,t),(d_k,T) \in A^k_r} x^k_{(i,t,d_k,T)} = 1 - y_k \quad \forall k \in K
\]

(6)

\[
\sum_{k \in K} \sum_{(i,t,j,t') \in B(i,t,j,t')} x^k_{(i,t,j,t')} \leq 1 \quad \forall (i,j) \in L
\]

(7)

\[
\sum_{k \in K} \sum_{(i,t,j,t') \in A_{t',i} \cap \{t \neq j, t' \leq t''\}} x^k_{(i,t,j,t')} + \sum_{(i,t,j,t') \in A_{t',i} \cap \{t \neq j, t' \geq t''\}} x^k_{(i,t,j,t')} - 1 + y_k \leq \text{Cap}_i
\]

\forall i \in \mathbb{N}, t'' \in T

(8)

\[
x^k_{(i,t,t')} \geq t^\text{wait}_{k,i} - T \times y_k \quad \forall k \in K, i \in \mathbb{N}
\]

(9)

\[
y_k = 0 \quad \forall k \in K : \text{dep}_{p_k,o_k} \leq H^\text{start}_d
\]

(10)

\[
x^k_{(i,t,j,t')} \in \{0, 1\} \quad \forall (i,t,j,t') \in A_{tr}, k \in K
\]

(11)

\[
y_k \in \{0, 1\} \quad \forall k \in K
\]

(12)

Passenger route choice constraints:

\[
\sum_{(o'_p,e'_p,j,t') \in A^p_{pa}} c^p_{o'_p,e'_p,i,j,t'} = D_p \quad \forall p \in P
\]

(13)
\[
\sum_{(i,t,j,t') \in A_{pa}^p} v^p_{(i,t,j,t')} = D_p \quad \forall p \in P
\] (14)

\[
\sum_{(i,t,j,t') \in A_{pa}^p; \ i \neq d'_p,d''_p} v^p_{(i,t,j,t')} = \sum_{(j,t';i,t) \in A_{pa}^p; \ i \neq d'_p,d''_p} v^p_{(j,t',i,t)} \quad \forall p \in P
\] (15)

\[
\sum_{p \in P} v^p_{(i,t,j,t')} \leq \sum_{k \in K} q_k \times x^k_{(i,t,j,t')} \quad \forall (i,t,j,t') \in A_{tr} \cap A_{pa}^p
\] (16)

\[
v^p_{(i,t,j,s)} \geq D(p) \times x^k_{(i,t,j,s)} \quad \forall k \in K, p \in P^k, (i,t,j,s) \in A_{tr}^k \cap A_{pa}^p
\] (17)

\[
0 \leq v^p_{(i,t,j,s)} \leq D(p) \quad \forall p \in P, (i,t,j,s) \in A_{pa}^p
\] (18)

In the objective, we minimize the total general passenger travel cost. Constraints (4), (5) and (6) are the train flow conservation constraints. Constraint (7) prevents any two conflict arcs being occupied by trains simultaneously. This is also called the headway constraint for any two trains. Constraint (8) is the station capacity constraint, which denotes that the total number of trains in a station \(i\) at any time \(t''\) cannot exceed the station capacity. Constraint (9) ensures that the dwell time of each train \(k\) at each intermediate station \(i\) should be no less than the scheduled dwell time, where parameter \(t_{wait}^{i,k}\) is the planned dwell time of train \(k\) at station \(i\). However, if a train is canceled, this constraint does not need to be respected any more. Constraint (10) denotes that if train \(k\) depart from its origin before the occurrence of the disruption, it cannot be canceled. This is due to the fact that no anticipation on the disruption is allowed. The last two constraints show the domain of binary variables \(x^k_{(i,t,j,t')}\) and \(y_k\).

In the passenger route choice constraints, equations (13), (14) and (15) denote the passenger flow conservation. Train capacity constraint is given by (16), which denotes that the total number of passengers that use arc \((i,t,j,t')\) cannot exceed the total capacity of train \(k\) that utilizes the same arc. It also combines the train scheduling variable \(x^k_{(i,t,j,t')}\) with passenger routing variable \(v^p_{(i,t,j,t')}\), which means that if passengers choose to run on arc \((i,t,j,t')\), at least one train must use \((i,t,j,t')\). Constraint (17) ensure that if train \(k\) is not canceled, all passengers in group \(p\) which have booked tickets for train \(k\) will keep using train \(k\). Note that this constraint can be relaxed if passengers are allowed to change trains freely, especially when no seat reservation system is applied. The last constraint shows the domain of variable \(v^p_{(i,t,j,s)}\).

5 Experiments and Results

In this section, we demonstrate our experimental results based on the Beijing-Shanghai high speed railway line in China. IBM ILOG CPLEX 12.8 was utilized as the solver with CPLEX parameters set to their default values. All our experiments are ran on an Intel(R)Core(TM)i7-7700 Processor CPU @3.60GHz 3.60GHz, 16.0GB RAM desktop.
5.1 Test instance and parameter setting

To test our model, the Beijing-Shanghai high-speed railway line is applied. This line with a length of 1318km, and it is quite busy. For more detail information about this line, we refer to Zhan et al (2016). We will select four trains that run between Beijing South Station and Shanghai Hongqiao Station after the disruption occurs.

The original timetable was used for the year 2013. Train running time in each segment is assumed to be the same as scheduled. The minimum dwell time of a train in each station are based on the original timetable. The arrival and departure headway time between two consecutive train services in each station in the same direction is set to 3 minutes. The track release time in a station is assumed to be 1 unit of time, i.e., 1 min in our experiment. If a train service dwells at an intermediate station, the minimum dwell time equals the scheduled dwell time. The station capacities for intermediate stations from Beijing South to Shanghai Hongqiao are the same as those in Zhan et al (2015). They are 1, 2, 2, 2, 4, 2, 1, 2, 2, 3, 2, 1, 2, 2, 1, 1, 2, 2, 2 and 2 respectively. Since we only consider the long distance trains between the mentioned two stations, we assume half of the total station capacity is used by these trains (Zhan et al (2015)). The penalty for canceling a high speed train is equal to 5000, and the penalty for delaying a high speed train for 1 min is 5, which are the same as those in Zhan et al (2015). We only consider 4 trains after the disruption occurs as a small example to illustrate our model.

We assume that a disruption occurs in the segment between Dezhou East and Jinan West station at 12:00 in the morning. We assume that the maximum delay time of a train arrival or departure is the duration time of the disruption plus half an hour. This assumption is enough to ensure trains select the proper running and dwell arcs in the space-time network. After the disruption, trains can continue their journey. Thus, they will not be delayed for a long time after the disruption.

5.2 Computational results

To check whether our space-time network based train rescheduling model works, we compared our current model with the train rescheduling model in paper Zhan et al (2015). We use the objective function in paper Zhan et al (2015) as the objective in the current model but in a space-time form. In the objective function, we minimize the total weighted train delay and cancelation. Only the train operational constraints in the current model is considered, since in the previous paper we only reschedule trains from operator’s point of view. We assume that the duration of the disruption is known in advance. Thus, trains can expect the end time of the disruption, and can start from a stopping station a short time before the end of the disruption.
5.2.1 From operator’s perspective

Based on the train rescheduling model (formula (4) \( \sim (12) \)), the values of parameters in section 5.1, and the assumed disruption scenario, the rescheduling results for trains are shown in Table 3. In Table 3, the first column is which model is applied. The second column shows the total number of trains considered. The objective value is shown in column 3; The total number of canceled trains is given in columns 4. The last two columns are the computation time and gap. Note that gap here indicates the difference between the current obtained upper bound and lower bound.

Table 3 Computational results for trains under strategy DTWS

<table>
<thead>
<tr>
<th>Research</th>
<th>Train No.</th>
<th>Objective</th>
<th>cancellation</th>
<th>Computation time (s)</th>
<th>Gap (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhan et al (2015)</td>
<td>4</td>
<td>27670</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>The current research</td>
<td>4</td>
<td>27670</td>
<td>0</td>
<td>212</td>
<td>0</td>
</tr>
</tbody>
</table>

From Table 3, we can see that the results obtained by the model in the current paper and those obtained by the model in Zhan et al (2015) are the same for the same disruption scenario. The objectives are exactly the same. However, the computation time of the current space-time based model is much slower for this small case due to the fact that the current model has more variables when adding the time dimension. But the space-time based model is prior to the previous event-activity based model in modeling the passenger route choice. The original timetable for the small example is given in Figure 2. In the figure, each line with a special color is a train line with the train number behind it. The yellow rectangle is the disruption area. All these hold for other timetable pictures below. The rescheduled timetable for the disruption scenario shows in Figure 3.

5.2.2 From passenger’s perspective

Based on the passenger-oriented rescheduling model (formula (3) \( \sim (18) \)), we also test it on the same case. We consider train rescheduling and passenger route choice simultaneously. We compare results for two approaches. One is a sequential approach where train timetable is first rescheduled based on the operator’s perspective model mentioned in the previous section, and then the passengers are assigned based on the obtained timetable. The other approach is the integrated approach introduced in the current paper where train rescheduling and passenger assignment are done simultaneously. The weight for each type of passenger arcs are given in Table 4.

The passenger ODs for these 4 trains are assumed as in Table 5. Each train has 3 ODs, and there are 12 ODs in total. The maximum capacity of a train is assumed to be 150. Based on these parameter values, the results for the
disruption scenario are shown in Table 6. It can be seen that if we reschedule trains and passenger route choice simultaneously, the result is better, see the objective values in Column 3. In addition, comparing the disposition timetables in Figures 3 and 4, we can see that train 2 and train 3 change order after the disruption in the timetable obtained by integrated approach. This is because more passengers (80) in train 2 than those in train 3 (40) after station 5. In principle, priority should be given to train 2 to minimize the total passenger travel cost. Although, the computation time for the space-time network based integrated approach is much longer, it can formulate the integrated train rescheduling and passenger route choice problem in a linear form which is non
Table 4  Values of weighting factors in the passengers' generalized travel time

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>2.5</td>
<td>[min/min]</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>10</td>
<td>[min/transfer]</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>1</td>
<td>[min/min]</td>
</tr>
</tbody>
</table>

linear if we extended the event-activity model in Zhan et al (2015). Besides, the space-time network based model is better for problem decomposition for large scale real-world problems, which will be discussed in our future research.

Table 5  Assumed passenger OD

<table>
<thead>
<tr>
<th>OD</th>
<th>Train No.</th>
<th>Origin</th>
<th>Destination</th>
<th>Passenger volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>23</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>23</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>1</td>
<td>23</td>
<td>40</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>1</td>
<td>23</td>
<td>80</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 6  Computational results from passenger’s perspective

<table>
<thead>
<tr>
<th>Approach</th>
<th>Train No.</th>
<th>Objective cancelation</th>
<th>Computation time (s)</th>
<th>Gap (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential</td>
<td>4</td>
<td>158815</td>
<td>0.2 + 0.34</td>
<td>0</td>
</tr>
<tr>
<td>Integrated</td>
<td>4</td>
<td>158640</td>
<td>7402</td>
<td>0</td>
</tr>
</tbody>
</table>

To demonstrate the passenger route choice in a disrupted situation, we assume that train 1 is require to be canceled due to the limited track capacity or high train operational cost. Therefore, according to our model, passengers booked the tickets of train 1 have to change to other trains. From our results, total 50 passengers on train 1 from station 1 to station 23 transfer to train 2 since the spare capacity of train 2 is 50. And the next 30 passengers on train 1 from station 1 to 23 transfer to train 3. The 10 passengers on train 1 both from station 1 to station 5 and from station 1 to station 4 are transfer to train 3. From Figure 6, we can see that train 2 with 150 passenger has change to
The rescheduled timetable for integrated approach provides a better place than train 3 with only 70 passenger after station 5. Therefore, the objective value obtained by the integrated approach is better.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Train No.</th>
<th>Objective</th>
<th>cancelation</th>
<th>Computation time (s)</th>
<th>Gap (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential</td>
<td>3</td>
<td>157120</td>
<td>1</td>
<td>0.2 + 0.85</td>
<td>0</td>
</tr>
<tr>
<td>Integrated</td>
<td>3</td>
<td>156825</td>
<td>1</td>
<td>10800</td>
<td>1.3</td>
</tr>
</tbody>
</table>

6 Conclusion

Real-time train rescheduling in a complete blockage is quite important for train dispatchers and passengers. In this research, we have extended the train rescheduling model in Zhan et al. (2015) to include the passenger route choice. The integrated train rescheduling and passenger choice problem is formulated by an ILP model based on a space-time network. We have tested our model on a small Chinese case to illustrate that reschedule trains and passengers simultaneously can obtain a better disposition timetable from passenger’s perspective. In addition, our approach can guide the disrupted passengers to select a better route to their destination.

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