A train rescheduling algorithm which minimizes passengers’ dissatisfaction based on MILP formulation

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Abstract Train traffic is sometimes disrupted and train rescheduling is performed by dispatchers in that case. To help them, train rescheduling algorithms, particularly timetable rescheduling algorithms have been rigorously studied. Passenger-oriented approaches such as minimizing the total arrival delay of passengers at their destinations are said to be important, but they require passenger origin-destination data as input, which are not necessarily available. To this issue, a unique approach which does not require such data but minimizes the total number of situations in a timetable which would breed dissatisfaction among passengers by simulated annealing has been proposed. In this paper, we present a Mixed Integer Linear Programming (MILP) formulation to this problem. By modeling as MILP, we can avoid difficulty of defining a good neighborhood in the simulated annealing framework.

Keywords rescheduling · algorithm · MILP · disruption · railways
1 Introduction

In Japan, railways play the most significant role in both urban and intercity transportation. In fact, trains are operated every couple of minutes in many cities carrying a massive number of commuters, and even in the high speed railway lines hundreds of trains a day are operated every three to four minutes. Thus, it is strongly desired for railways to provide passengers with a stable and reliable transportation service.

Although Japanese railways are known to be very punctual, train traffic is sometimes disrupted when accidents, natural disasters, engine troubles, etc. happen. In order to restore the disrupted traffic, a series of modification to the current train schedule is done. This task is called “train rescheduling” (refer to Hansen and Pachl (2014)) and is performed by train dispatchers. Recently, computer systems which help train dispatchers began to be put into a practical use. These systems, however, are lacking in functions to automatically make rescheduling plans. Hence, train rescheduling is totally left to the dispatchers, and this is a heavy burden for them.

In order to break through such a situation, it is required for train rescheduling systems to be equipped with an advanced function of automatic rescheduling. To make train rescheduling plans, however, is an extremely difficult task since objective criteria of rescheduling plans are diverse depending on the situations, it is a large-sized and/or complicated combinatorial problem in which sometimes hundreds or thousands of trains are involved, an urgent problem solving is required, etc. In terms of the rescheduling criteria, it is very important to include passengers’ viewpoints.

Train rescheduling includes timetable rescheduling, rolling-stock rescheduling and crew rescheduling. Among them, the first one has been most rigorously studied, and there have been a large number of timetable rescheduling studies in the rail transport rescheduling literature (refer to Törnquist (2006); Cacchiani et al (2014); Corman and Meng (2015); Fang et al (2015)). Timetable rescheduling consists of a combination of actions called “rescheduling measures” which changes the original/current timetable. What rescheduling measures are applied depends on the scale and the situation of disruption. Technical Research Committee on Advanced Rail Transport Planning and Management, The Institute of Electrical Engineers of Japan (2010) presents various rescheduling measures and disrupted situations where each of them is applied in Japan. Based on this work, we distinguish between general and detailed rescheduling measures. The former measures are first implemented when the disruption is large. They include cancellation of trains, global rerouting of trains which is also called a detour to avoid the disrupted area, and shuttle operations between undisrupted areas. The latter measures are performed when the disruption is small or certain general rescheduling measures have already been implemented. They include reordering of trains such as change of departing orders, local rerouting of trains inside a station (change of tracks in a station), retiming of trains (adjustment of trains’ interval), change of train
types and change of rolling stock assignment to trains (change of train-set operation schedules).

On general rescheduling measures, the application of a predetermined emergency timetable (Corman and D’Ariano (2012)) or rescheduling pattern (Nakamura et al (2011)) are studied. Louwerse and Huisman (2014) and Veelenturf et al (2016) take an optimization approach. On detailed rescheduling measures, train-punctuality-oriented approaches such as minimizing the total delay of trains have been much studied. By contrast, there are passenger-oriented approaches such as minimizing the total arrival delay of passengers at their destinations. Among them, “delay management” is a famous concept (initial work is covered by Schöbel (2006)), and research on this topic has been advancing (refer, for instance, to Dollevoet et al (2012)). Note that similar ideas are proposed, for instance, by O’Dell and Wilson (1999); Suhl et al (2001). These approaches take passengers’ behavior, or a train choice by passengers, into consideration. Timetable modification and passenger rerouting which depends on the modified timetable are simultaneously handled by Sato et al (2013); Dollevoet et al (2015). Since the integrated model is a huge optimization problem, iterative approaches between timetable modification and passenger rerouting are proposed by Kanai et al (2011); Dollevoet et al (2014); Corman et al (2017).

Delay management or similar approaches require passenger origin-destination (OD) data as input. These data, particularly real-time ones in a disrupted situation are not necessarily available for all railway lines or networks. To this issue, Tomii et al (2005) presents a unique approach. It does not explicitly deal with passengers’ behavior but situations in a timetable which would breed dissatisfaction among passengers are sorted out in advance. It then produces a rescheduling plan which minimizes the total number of such situations by simulated annealing. It applies both general and detailed rescheduling measures: cancellation of trains, train reordering, local rerouting, train retiming and change of rolling stock assignment. This makes the definition of a neighborhood in the simulated annealing and therefore providing a good rescheduled timetable very difficult.

In this paper, inspired by Tomii et al (2005), we present a Mixed Integer Linear Programming (MILP) formulation of a timetable rescheduling problem. We provide a rescheduled timetable so that the total number of situations which would breed dissatisfaction among passengers in that timetable is minimized. By modeling as MILP, no neighborhood is required. Another advantage of our model is that our model is smaller than other passenger-oriented MILP models which have variables on passengers’ behavior like Sato et al (2013).

This paper is organized as follows; in section 2, we will explain an outline of the timetable rescheduling problem together with explanations about why timetable rescheduling is difficult and how we should evaluate the result of timetable rescheduling. In section 3, we introduce our basic idea to evaluate a result of timetable rescheduling based on dissatisfaction of passengers. Then in section 4, we explain our MILP-based algorithm which minimizes passengers’ dissatisfaction. In section 5, we show some results of numerical experiments.
followed by discussions. Conclusions and future work are presented in Section 6.

2 Timetable rescheduling

2.1 What is timetable rescheduling?

When train traffic is disrupted, train dispatchers try to reschedule the disrupted traffic. Major rescheduling measures used in rescheduling are as follows:

- (partial) cancellation of trains
- change of tracks in a station
- change of departing orders of trains
- change of train-set operation schedules
- adjustment of trains’ interval
- change of train types

Fig. 1 illustrates a change of departing orders of trains. In railway lines which connect a suburban area and a downtown area, express trains are operated to offer a fast transportation service for passengers who live distant from the downtown area. In order to give a good service to customers of small stations where express trains do not stop, we usually offer a timetable which is called “coupling” between express and local trains. A change of departing orders works effectively for such a timetable when there occurs a delay. In Fig. 1, an original schedule is drawn in light gray and we assume that the departure of the express train (Train X) from Station A is delayed. Fig. 1 left is a result when no change of the schedule was performed. The local train (Train Y) must wait for the express train (Train X) in vain. Fig. 1 right is a result when the departing orders of the trains at Station B was changed so that Train Y departs first to avoid for Train Y to wait for Train X at Station B.

![Fig. 1 Change of departing order](image-url)
In Fig. 2, we show a rescheduling technique called “store and new train.” Let us assume that the departure of Train X from Station A is delayed. This delay will be propagated to the turning back train which departs from Station C. In this case, a useful technique to reduce the delay is to set up a new train from Station B as Train X (we assume here that there exists a depot at Station B), to give up operation of the original Train X at Station B and to store the train-set at the depot. Thus, the delay of Train X is not propagated to the turning back train as shown in Fig. 2 right.

Another example of a combination of rescheduling measures is turning back of trains before their destinations. It is a combination of partial cancellation of trains and change of train-set operation schedules (and possibly, change of tracks in a station). This technique is very often used when the time of disruption is expected to become long.

2.2 Why timetable rescheduling is so difficult?

Although we showed only simple examples in the previous subsection, we have to note that a series of the rescheduling measures are successively applied in actual situations. Timetable rescheduling is known to be a quite difficult work. Major reasons for this are as follows:

- It is difficult to decide an objective criterion (or criteria). Criteria for rescheduling differ depending on various factors such as severity of accidents, time when the accident occurred, characteristics of the line such as whether it is a commuter line or an intercity railway line, etc. Although delays of trains are usually considered to be undesirable, regaining the schedule is not so significant in railway lines where trains run with short
intervals such as trains in urban areas. It is considered to be far more important to prevent the intervals from becoming too large. The criteria should be even different depending on the time when an accident has happened. During rush hours in the morning, to keep the constant intervals between trains and to maintain necessary transportation capacity are considered to be more important than to reduce delays, whereas in the afternoon it is most important to regain the schedule before evening rush hours and sometimes we should cancel a considerable number of trains since trains in the afternoon are less congested. Another example is that when the accident is serious, passengers will not care about small delays but complain very much if trains do not come for a long time. On the other hand, if the accident is a small one, they will complain even for the delay of several minutes.

- Train rescheduling is a large-sized combinatorial problem. In urban areas, the number of trains involved often reaches hundreds or even thousands. Moreover, in Japan, train schedules are prescribed by a unit of 15 seconds (in urban lines, the time unit is five seconds). In making train rescheduling plans, we have to determine departure/arrival times and tracks for each train, whether to cancel trains or not, etc. As a matter of fact, when trains are delayed about one hour, the number of required schedule modification sometimes reaches several hundreds.

- A high immediacy is required. Since train rescheduling plans are made in order to modify the schedule of trains which are running at that time, they have to be made quickly enough.

- All the necessary information cannot always be obtained. In particular, if we would like to make a rescheduling plan based on precise passenger behavior, it becomes necessary to get information about how crowded trains are/will be, how many passengers are/will be waiting for trains at stations, how many passengers will emerge at stations, etc. Under current technology, however, it is quite difficult or almost impossible to get or estimate such information.

2.3 Timetable rescheduling algorithm

A timetable rescheduling algorithm is an algorithm to generate an optimal, near-optimal or feasible timetable rescheduling plan. The algorithm receives the current timetable, information on facilities such as track layouts and information about the disruption (e.g., when the disruption has appeared and expected duration of the disruption together with current delays of trains if they exist) as input. It then outputs a rescheduling plan which minimizes some objective function by applying a series of rescheduling measures.
2.4 Criteria of timetable rescheduling

In order to regard the train rescheduling problem as a combinatorial optimization problem and to develop a timetable rescheduling algorithm, we first have to clarify the objective function of the problem. Until now, the following ideas are proposed:

- (The total sum of) delay time of trains should be minimized.
- The number of cancelled trains should be minimized.
- Time required until the train traffic is normalized should be minimized.
- Passengers' disutility should be minimized.
- The gap of service levels between one which passengers expect and one passengers actually receive should be minimized.

None of these criteria, however, are satisfactory. An idea to use delay time of trains is not appropriate when a number of trains are cancelled. The more trains are cancelled the less the delay would be, but passengers suffer from inconvenience since trains are crowded and frequency of trains decreases. In addition, significance of punctuality might be different from one train to another train. For instance, there might be a train which has a connection to another train. In that case, the delay of this train should be smaller than the minimum time required for the connection. It is not necessary to minimize the delay but if the delay is larger than the threshold, the connection must be lost and passengers will complain. Let us show another simple example in Fig. 3.

We assume that the departure of an express train (Train 4) is delayed seven minutes (original timetable is shown in gray line). In Fig. 3 left, no change is given and the total sum of arrival delays at Station A is 14 minutes. On the other hand, Fig. 3 right is a result of rescheduling, in which the departing order of trains at Station B was changed and this makes the total sum of the arrival delays at Station A eleven minutes. So, if we consider only the (total sum of) delays we may conclude that Fig. 3 right is better. Assume here that the express train (Train 4) has a connection to another train at Station A which is supposed to depart nine minutes after the arrival of the express train in an original timetable. In this case, if we adopt Fig. 3 right, many passengers who wanted to catch their next train at Station A will be dissatisfied and may complain since the connection is broken.

The idea to use the number of cancelled trains as a criterion has an opposite problem. Although it is true that cancellation of trains often inconveniences passengers, this is the most effective method to restore the disrupted schedule. Thus, it is often desirable to cancel an appropriate number of trains and to normalize the schedule particularly when an accident happened before rush hours.

Passengers' disutility and the gap of service levels seem to be promising as the criteria from passengers' viewpoint, but they are quite difficult to measure with existing technology primarily since it is quite difficult to get or estimate the time-dependent (future) OD data which play a key role to estimate passengers' disutility.
3 Evaluation of timetable rescheduling from the viewpoint of passengers’ dissatisfaction

From the discussions in the previous sections, we learned that we need to devise criteria which are applicable to different situations and are possible to compute very quickly using data we can easily obtain. In this paper, we propose to use “passengers’ dissatisfaction” as the criteria. We show a list of situations in Table 1 where we believe that passengers will be dissatisfied:

<table>
<thead>
<tr>
<th>Type</th>
<th>content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival delay</td>
<td>delay of an arrival of a train exceeds a threshold.</td>
</tr>
<tr>
<td>Departure delay</td>
<td>delay of a departure of a train exceeds a threshold.</td>
</tr>
<tr>
<td>Waiting time</td>
<td>interval between trains exceeds a certain threshold.</td>
</tr>
<tr>
<td>Dwell time</td>
<td>increase of a dwell time of a train exceeds a threshold.</td>
</tr>
<tr>
<td>Running time</td>
<td>increase of a running time of a train exceeds a threshold (it often occurs when a train is kept waiting before it arrives at a station since its scheduled stop track is being occupied by another train).</td>
</tr>
<tr>
<td>Connection</td>
<td>connection is lost.</td>
</tr>
</tbody>
</table>

We first scrutinize in what cases passengers would be dissatisfied considering conditions such as severity of accidents, characteristics of railway lines, etc. Then these cases are accumulated in a file we call “Dissatisfaction File.” We can prepare several Dissatisfaction Files each of which corresponds to a different situation. Before our rescheduling algorithm starts, it chooses the most suitable Dissatisfaction File.
A weight is put to each dissatisfaction taking its content such as the amount of delays, the number of possible passengers, etc. into account. We calculate a weighted sum of each dissatisfaction contained in a rescheduling plan. We call this value “dissatisfaction index.” From an alternative view, passengers’ dissatisfaction defined above can be regarded as “constraints” to be satisfied. In this sense, we can say that we treat the timetable rescheduling problem as a sort of a constraint optimization problem to find a schedule which satisfies the constraints defined in the Dissatisfaction File as much as possible.

4 A timetable rescheduling algorithm which minimizes passengers’ dissatisfaction based on MILP formulation

We introduce a timetable rescheduling algorithm which minimizes passengers’ dissatisfaction. We model the timetable rescheduling problem as a MILP problem and solve it by an optimization solver. As criteria of the optimization, we use passengers’ dissatisfaction explained in Section 3.

In this paper, we assume that the track layout is like one shown in Fig. 4; the track is double and we do not consider bi-directional operation. We also assume that there exist stations where train can turn back such as the fourth station from left. There also exist stations which have only one track for each direction such as the second and the third stations from left. Obviously, train cannot turn back at these stations and in the formulation that we show below, we ignore these stations since these stations are usually not so important.

Fig. 4 Track layout

In the following subsections we show our formulation. We omit, however, notations, variables and constraints concerning train operation in this paper due to the limitation of the space. They are in principle the same as those by Imada and Tomii (2017). We also omit them for upward trains since they are almost the same as those for downward trains.

4.1 Notations

We introduce here sets and their elements commonly used in our timetable rescheduling MILP formulation.
Suffix
s: Station id
s_{first}: First station for downward trains
s_{final}: Final station for downward trains
t: Train id
s_{Next}(t, s): Station next to Station s for Train t
dtype: Type of passengers’ dissatisfaction

Set
S: Set of stations
S_{con}: Set of stations at which a passenger connection can be done
T_{down}: Set of downward trains
Dtype: Set of passengers’ dissatisfaction types

Constants
A^s_t: Planned arrival time of Train t at Station s
D^s_t: Planned departure time of Train t at Station s
\alpha^s_{t,dtype}: Threshold on passengers’ dissatisfaction of Type dtype at Station s for Train t
dweight^{dtype}: Weight of passengers’ dissatisfaction for Type dtype
M: Big value

Variables
a^s_t: Rescheduled arrival time of Train t at Station s
d^s_t: Rescheduled departure time of Train t at Station s
d^s_{Next(t, s)}: Rescheduled departure time of Train t’s next train at Station s
D^s_{Next(t, s)}: Planned departure time of Train t’s next train at Station s
d^s_{Con(t, s)}: Rescheduled departure time of Train t’s connecting train at Station s
v^s_{t,dtype} := \begin{cases} 1 & \text{if Train } t \text{ does not satisfy Constraint dtype at Station } s \\ 0 & \text{otherwise} \end{cases}

4.2 Constraints

We formulate the situations where passengers will be dissatisfied according to Table 1 as follows.

Dissatisfaction for arrival delay:
\[ M \times v^s_{t,ad} \geq (a_t^s - A_t^s) - \alpha^s_{t,ad} \]
\[ \forall t \in T_{down}, \ \forall s \in S \setminus \{s_{first}\}. \]
Constraint (1) expresses that the arrival delay (abbreviated as $a_{d}$) of Train $t$ at Station $s$ should be smaller than the threshold $\alpha_{t,ad}$.

Dissatisfaction for departure delay:

$$M \times v_{t,dd}^s \geq (d_t^s - D_t^s) - \alpha_{t,dd}^s$$

$$\forall t \in T_{down}, \ \forall s \in S \setminus \{s_{final}\}.$$  \hspace{1cm} (2)

Constraint (2) expresses that the departure delay (abbreviated as $a_{d}$) of Train $t$ at Station $s$ should be smaller than the threshold.

Dissatisfaction for dwell time:

$$M \times v_{t,dt}^s \geq ((d_t^s - a_t^s) - (D_t^s - A_t^s)) - \alpha_{t,dt}^s$$

$$\forall t \in T_{down}, \ \forall s \in S \setminus \{s_{first}, s_{final}\}.$$  \hspace{1cm} (3)

Constraint (3) expresses that the dwell time (abbreviated as $a_{d}$) of Train $t$ at Station $s$ should be smaller than the threshold.

Dissatisfaction for running time:

$$M \times v_{t,rt}^s \geq ((d_{t_{next}}^s - a_t^s) - (D_{t_{next}}^s - D_t^s)) - \alpha_{t,rt}^s$$

$$\forall t \in T_{down}, \ \forall s \in S \setminus \{s_{final}\}.$$  \hspace{1cm} (4)

Constraint (4) expresses that the increase of the running time (abbreviated as $a_{r}$) of Train $t$ from Station $s$ to its next station should be smaller than the threshold.

Dissatisfaction for frequency:

$$M \times v_{t,fi}^s \geq ((d_{t_{next}}^s - a_t^s) - (A_{t_{next}}^s - D_t^s)) - \alpha_{t,fi}^s$$

$$\forall t \in T_{down}, \ \forall s \in S \setminus \{s_{final}\}.$$  \hspace{1cm} (5)

Constraint (5) is about a waiting time. The interval (abbreviated as $i$) between the departure of Train $t$ and the that of the next train at Station $s$ should be smaller than the threshold.

Dissatisfaction for connection:

$$M \times v_{t,con}^s \geq (d_{t_{con}}^s - a_t^s) - \alpha_{t,con}^s$$

$$\forall t \in T_{down}, \ \forall s \in S_{con}.$$  \hspace{1cm} (6)

$$-M \times v_{t,cu}^s \leq (d_{t_{con}}^s - a_t^s) - \alpha_{t,cu}^s$$

$$\forall t \in T_{down}, \ \forall s \in S_{con}.$$  \hspace{1cm} (7)

Constraints (6) and (7) express that the interval (abbreviated as $c1, cu$) between the arrival time of Train $t$ and the departure time of its connecting train at Station $s$ should be smaller and larger than the thresholds, respectively.
4.3 Objective Functions

Although we are primarily interested in minimizing passengers’ dissatisfaction, we prepare below three objective functions for comparison.

Objective function to minimize the total sum of arrival delays:

\[
\min \sum_{t \in T_{\text{down}}, s \in S \setminus \{s_{\text{first}}\}} (a_t^s - A_t^s). 
\] (8)

Objective function to minimize (only) passengers’ dissatisfaction:

\[
\min \sum_{t \in T_{\text{down}}, s \in S} \sum_{k \in D_{\text{type}}} v_t^s \times d_{\text{weight}}^k. 
\] (9)

Objective function to minimize the weighted sum of passengers’ dissatisfaction and the total sum of arrival delays:

\[
\min \sum_{t \in T_{\text{down}}, s \in S \setminus \{s_{\text{first}}\}} (a_t^s - A_t^s) + \sum_{t \in T_{\text{down}}, s \in S} \sum_{k \in D_{\text{type}}} v_t^s \times d_{\text{weight}}^k. 
\] (10)

5 Numerical experiments and discussions

We have implemented our algorithm on ILOG CPLEX Optimization Studio 12.7.1 (offered by IBM (2018)) and have conducted numerical experiments.

5.1 Analysis on the objective functions

We have conducted numerical experiments using a simple timetable for the three objective functions shown in Subsection 4.3. We have learned that when we use the objective function (9), sometimes we get an unreasonable result. We will give an example. In the timetable of Fig. 3, we assume that passengers will be dissatisfied if a connection is lost at Station B and we set a constraint about the connection concerning the arrival of Train 2 and the departure of Train 4 must be less than a certain threshold. If we do not set any other constraints about passengers’ dissatisfaction, the result we get when the departure of Train 4 is delayed, is that Train 2 departs from Station C on time but arrives late at Station B so that the constraint of the connection is satisfied. This result, however, is regarded to be quite unreasonable.

One idea to avoid such unreasonable result is, of course, to set other constraints such as a constraint about the arrival delay of Train 2 at Station B. Another idea might be to include a sum of delays in the objective function just like the objective function (10).

This comparison is very interesting, but to make things easy to understand, we adopt the objective function (10) in this paper as a preliminary analysis.
5.2 Numerical Experiments

In Fig. 5, we show the planned timetable.

![Fig. 5 Planned timetable](image1)

In Fig. 6, we show the result we have gotten by using the objective function (8) when we assume that the departure of Train 2 from Station A is delayed for eleven minutes. We can observe that other trains (Trains 1, 3, 5, 6) departs from Station A on time and in order to reduce departure delays at Station E, turn back of trains are changed: from Train 1 to Train 8 (originally from Train 1 to Train 9) and from Train 2 to Train 9 (originally from Train 2 to Train 8). In addition, by changing the departing orders of trains, particularly upward ones at Station C, the delays are reduced.

![Fig. 6 Rescheduling—delays](image2)
We assume that passengers will be very much dissatisfied with the delay of Train 9 and have given the constraint that the departure of Train 9 should not be delayed. We have adopted the objective function (10) and show the result in Fig. 7. Now, Train 9 departs on time and the constraint on the delay of the train is satisfied.

![Fig. 7 Rescheduling—passengers' dissatisfaction](image)

5.3 Discussions

From the numerical experiments, we have learned the followings:

- This algorithm tries to satisfy as many constraints as possible and if there are not enough ones in the Dissatisfaction File, this algorithm may produce a solution which seems impractical.
- It is of course a very interesting point how to specify constraints on passengers’ dissatisfaction. In addition, we should clarify if it is good to include delays in the objective function as we have done in our experiments. Another idea might be to set constraints to many spots, such as delays must be less than two minutes for all the trains at all the stations etc. It is an interesting topic to analyze the characteristics of these objective functions and to clarify the advantages and disadvantages of these approaches.
- It is also an important topic how to specify the weight for each constraint about passengers’ dissatisfaction. By setting constraints like “the weight is one for five minutes’ delay, five for ten minutes’ delay, 50 for 15 minutes’ delay, etc.” we may be able to deal with non-linearity on passengers’ dissatisfaction.
6 Conclusion

In this paper, we have proposed a timetable rescheduling algorithm which minimizes passengers’ dissatisfaction based on our MLIP formulation. We in advance collect situations in a timetable which would breed dissatisfaction among passengers and keep them in the Dissatisfaction File. Our algorithm, tries to produce a timetable rescheduling plan, in which the total number of situations written in the file will become a minimum. From the numerical experiments, we have proved that our algorithm is promising although there remain issues to be settled in the very near future such as setting proper objective weight values on each situation as well as those on the amount of delays.

References

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