Optimizing mixed-fleet bus scheduling under range constraint

Lu Li, Hong K. Lo*, Feng Xiao

Abstract This paper develops a formulation for the multiple depot (MD) vehicle scheduling problem with multiple vehicle types (MVT), including electric buses (EBs), under range and refueling constraints. A novel approach is developed to generate the feasible time-space-energy (TSE) network for bus flow and time-space (TS) network for passenger flow, where the range and refueling issues can be precisely addressed. We then introduce the external cost associated with emissions, and investigate the minimum total system cost to operators and passengers by scheduling the bus fleet and locating the refueling stations. The problem is formulated as a mixed integer linear program (MILP) to find the global optimal solution. We then verify the effectiveness of the approach by using a small bus service network.

Keywords: Bus scheduling · Mixed fleet · Electric bus · Driving range · Refueling/Charging

1 Introduction

Roadside pollution has attracted the attention of more and more people recently. It is reported by the European Commission that poor air quality causes more premature deaths than road accidents, responsible for 0.31 million premature deaths in Europe every year. In England in 2013, 11,490 deaths were caused by heavily NO2 pollution (European Environmental Protection Agency, 2016). Obviously, the serious consequence of roadside emissions is underestimated. To improve air quality at

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roadside, over 220 cities and towns in fourteen countries around Europe are operating or preparing for Low Emission Zones. This has led to subsidy policies to promote alternative energy vehicles, such as hybrid and electric buses, to reduce or remove tail-pipe emissions. For public transportation, the majority of the bus fleet are heavy-duty diesel vehicles, which produce a substantial amount of air pollutants (US EPA, 2008). Converting the bus fleet from diesel to alternative energy sources will hence bring in sizeable reductions in emissions. During the process, range constraints are one critical issue to be tackled. Buses travel high daily mileages, so the range concern arising from electric or alternative energy buses becomes an important issue. In any case, in the deployment of alternative energy buses, particularly electric buses, the range constraints must be duly incorporated in bus route assignment and scheduling.

The traditional bus-scheduling problem is to cover all the trips in the timetable with fixed travel times and start and end locations. The objectives are to minimize the bus fleet size and the operating cost. Bunte and Kliewer (2009) gave an overview on vehicle scheduling problems and discussed several modeling approaches. Earlier bus-scheduling studies seldom considered energy and emissions. The main idea was to minimize the fleet size. The conventional bus service, which has fixed service schedules run by a single bus type, is not cost-effective due to the variable demand densities. Hence, some studies were conducted to investigate the multi-vehicle-type bus scheduling problem (MVT-VSP), involving different vehicle types of diverse capacities for timetabled trips (Hassold and Ceder, 2012; Ceder et al., 2013; Kim and Schonfeld, 2014; Hassold and Ceder, 2014). It is worth noting that the majority of studies on VSP are trip-based. Only a few considered time-dependent passenger demand by using passenger waiting time to reschedule the service trips. Hassold and Ceder (2012) and Ceder et al. (2013) investigated the problem that how to make public bus services more attractive. Both of them aimed to minimize the passenger waiting time to improve public transport reliability. An and Lo (2015) and Lo et al. (2013) considered the passenger cost when they formulated the network design problem on transit and ferry services. In addition to fulfilling all the timetabled trips, this study seeks to find the most cost-effective design to balance the costs between the operator and the passengers via the waiting penalty, which will improve both the utilization of buses and increase the attractiveness of the services.

In recent years, more and more VSP studies considered clean-energy buses due to environmental and energy concerns. Due to range constraints and long recharging time, the approach to schedule the bus fleet is quite different from the conventional VSP. Li (2013) proposed a vehicle scheduling model for electric buses, as well as compressed natural gas (CNG), diesel, and hybrid buses, respectively, with the maximum route distance constraints. Zhu and Chen (2013) established a single depot vehicle scheduling model (SDVS) for electric buses, in which the charging time has a positive linear relationship with the corresponding period of bus mileage. Both of these studies considered a single fleet VSP and set the timetable for bus charging according to its maximum driving distance. Notice that using the travel distance to
determine the driving range is not appropriate, since energy consumption is not only
determined by distance, but also travel speed, passenger loading, gradient of the
terrain, battery temperature, and current battery life (Thein and Chang, 2014; Goeke
and Schneider, 2015), etc. In this paper, we incorporate energy consumption into the
bus flow network generation by considering the impacts of travel speed and bus
loading.

Locating refueling stations is another indispensable issue for incorporating range
constraints in VSP. He et al. (2013) proposed a macroscopic planning model to design
the optimal number of charging stations allocated to each metropolitan area without
considering the exact locations and capacities. Then they proceeded to optimize the
deployment plan of charging lanes for electric vehicles over a general network (Chen
et al., 2016). Lim and Kuby (2010) developed three heuristic algorithms to locate the
refueling stations for alternative-fuels using path-based demands. Schneider et al.
(2014) presented an electric vehicle routing problem, which considered setting a set
of available recharging stations beforehand. Besides, some novel recharging facilities
were also investigated by recent studies. Liu and Wang (2017) proposed a modeling
framework for locating multiple types of battery electric vehicle charging facilities,
who considered wireless static and dynamic charging in the decision procedure. In
this paper, we generate the bus flow network by considering a set of feasible candidate
refueling stations to tackle the charging station locating problem.

For the mixed fleet VSP concerning the alternative energy sources, Li and Head
(2009) established a bus-scheduling model to minimize the operating cost and excess
vehicle emissions involving CNG and Hybrid buses, while range constraint and
charging time were not taken into account in this study. Beltran et al. (2009) and
Pternea et al. (2015) focused on developing an efficient model to solve a
sustainability-oriented variant of the transit route network design problem. Both of
them assigned the electric vehicles to pre-determined routes without considering the
charging time issue. By using a set of given weights, the objective functions sought
to minimize user, operator and external costs. A model of transit design for a mixed
bus fleet was developed by Fusco et al. (2013) to compute the internal and external
costs during the bus lifetime. It introduced an electric bus fleet to operate some lines
of a transit corridor in an urban area. Charging facilities were considered in this paper
whereas the assignment of travel routes for each electric bus was not addressed.
Goeke and Schneider (2015) optimized the routes of a mixed fleet of electric and
diesel commercial vehicles to fulfill the customer demand. Routing model was
developed to design the travel routes for each electric vehicle. Yet the emission
problem is not taken into consideration in their paper. Actually, few studies have been
conducted on the mixed bus fleet scheduling problem with regards to the range and
refueling constraints, and emissions reduction simultaneously. This paper considers
not only the daily travel routes of two types of buses with different energy sources,
but also different bus capacities to handle the demand.
In our previous work, we considered the mixed bus fleet management and government subsidy scheme for early-retiring, purchasing, and routing problems based on the frequency-based approach (Li et al., 2015; Li et al., 2016). In this paper, we consider the problem based on the schedule-based approach, which can explicitly deal with the range and refueling issues, as well as some additional problems, such as locating the refueling stations, considering passengers route choices, etc. In most previous studies on electric bus scheduling, they used time-space (TS) network to develop non-linear programs to solve for approximate solutions. Since energy is not tracked in these studies, range and refueling (charging time) constraints cannot be formulated linearly. In this study, we incorporate the energy consumption state variable explicitly into our model, and develop a novel approach to generate a time-space-energy (TSE) network for bus flow which linearly and precisely addresses the range and refueling constraints in VSP. Besides, we also build a TS network for passenger flow which allows for detour travels. Based on these two feasible networks, we develop a mixed-integer linear program (MILP) to formulate the multiple depot (MD) VSP with multiple vehicle types (MVT) to achieve global optimality, referred to here as MD-MVT-VSP. We minimize the costs of both operators and passengers, as well as the external cost caused by emissions, by assigning schedules and trips to a mixed fleet of EBs and diesel buses (DBs). We also incorporate the problem of locating refueling stations into our VSP and optimize them simultaneously. Compared with the conventional VSP, the proposed MD-MVT-VSP has distinctive advantages. The main contributions of this paper are as follows:

1. We consider the mixed bus fleet scheduling problem with bus service coordination. By integrating bus service, passenger movement, and bus emissions into one model, the optimal bus scheduling scheme is determined;
2. We propose a novel approach to generate the feasible TSE network for bus flow by capturing energy consumption explicitly. The range and refueling issues are precisely tackled based on the TSE networks generated, as is the refueling station location problem; 
3. We formulate the VSP as an ILP to find the global optimal solution instead of relying on heuristics as adopted by most studies.

2 Problem Description and Model Formulation

2.1 Problem description

The multiple depot vehicle scheduling problem with multiple vehicle types (MD-MVT-VSP) in this paper can be stated as follows: For a given set of depots and transit routes, given travelling distances between all pairs of bus termini, find the minimum system cost by determining the bus routing and service schedule within the planning horizon. We only need to consider the starting and ending termini in the formulation without involving the intermediate stops, as each bus will finish a complete trip and will not switch to another trip midway. The system cost involves the cost to operators
and passengers, as well as the external cost associated with emissions. Unlike the traditional bus-scheduling problem, passenger-waiting and demand-loss costs are introduced so that not all demand has to be satisfied. MD-MVT-VSP seeks to balance the benefits between the operator and the passengers from the social welfare perspective.

A vehicle schedule is feasible if (1) each trip of the scheduled timetable is covered exactly once, (2) each vehicle performs a feasible sequence of trips where each pair of consecutive trips can be operated in sequence, (3) each vehicle has sufficient energy to travel the next trip and (4) each vehicle is put into use again when it finishes its refueling process. MVT-VSP considers a heterogeneous fleet of vehicles using alternative energies. Since each vehicle type has its own driving range, its energy consumption and energy capacity are recorded for each trip for determining the specific moment at which refueling has to take place at a refueling station to avoid en route stranding.

For simplicity, we make the following assumptions:

1. The number of buses started from each depot at the beginning of the planning horizon is equal to the number of buses ended at the same depot at the end of the planning horizon.
2. Each transit route consists of bi-directional transit lines between two termini. By setting two bus termini instead of one with zero distance between them, the proposed model is also applicable for circular lines.
3. The demand for each origin-destination (OD) pair is given, and its arrival pattern at each terminus follows a trip distribution which is known beforehand.
4. The refueling time of each vehicle type follows a refueling rule, either linear or non-linear, with respect to the cumulative energy consumption, and full energy is restored after each refueling.
5. The energy consumption from (to) the depot to (from) the first (last) trip is very small compared with the service trips.

2.2 Generation of the feasible network

2.2.1 Bus TSE network

We generate the feasible daily network for each vehicle type with a specific energy source to keep track of each bus’s energy consumption. In the TSE network of bus flow, each node represents a specific location at a specific time with a specific level of energy consumption. Time is divided into a set of intervals by a fixed time duration, and energy consumption is divided into a set of levels by an energy step. In other words, at each time interval of each bus terminus, there are a set of nodes demarcated by different energy consumption levels. By keeping track of the energy consumption at each node, the remaining driving range can be calculated. A network is then built by connecting the arcs of all possible routes of the concerned bus type. In this way, not only do such TSE bus networks ensure that all buses travel within their energy limits, revealing refueling needs, they also allow for the incorporation of mixed-route bus trips.
Let \( I = \{1, 2, \ldots, i\} \) be the set of bus types with different energy sources. The TSE network is defined by a graph \( G(V^b, A^b) \) as shown in Fig. 1. \( V^b \) represents the set of nodes with \( V^b = O^b \cup T^b \cup F \), where \( O^b \) denotes the set of depots, \( T^b \) the set of time-expanded bus termini, and \( F \) the set of time-expanded refueling stations. \( A^b \) is the set of arcs with three subsets: service arc set \( S^b \), waiting arc set \( W^b \), and deadheading arc (the movement of a vehicle to a destination without serving any passenger) set \( D^b \), such that \( A^b = S^b \cup W^b \cup D^b \). Each service arc indicates a direct service trip between an OD pair starting at a specific time with a specific level of energy consumption, whereas each waiting arc connects two consecutive time nodes of the same terminus while maintaining the same energy level. Each deadheading arc represents a trip (i) from or to a depot, or (ii) to the starting terminus of another service trip, or (iii) to a refueling station.

Two parameters are introduced to adjust the travel time and energy consumption. One is \( \gamma_{jk}^v \), the adjustment parameter for speed related to the time of day when traveling on the arc \((j, k)\). The other is \( \gamma_{jk}^m \), the adjustment parameter for vehicle mass related to the passenger loading or demand. The travel time and energy consumption on each arc are determined by the travel distance \( d_{jk} \), the average travel speed \( v \), and the energy consumption rate \( \eta_i \) alongside with the two adjustment parameters. We define energy consumption as the cost of the arc \((j, k)\), i.e., \( q_{jk}^i = d_{jk}^i \eta_i \gamma_{jk}^v \gamma_m^i \), whereas travel time is denoted by \( t_{jk} = \frac{d_{jk}}{v \gamma_{jk}^v} \). For bus type \( i \), let \( Q_i \) be the energy capacity, \( \tau_{j}^i \) be the time of arrival at node \( j \), and \( e_i^j \) the cumulative energy consumption upon arrival at node \( j \). Finally, we define a set of indicators \( U = \{u_{jk}^i\} \) to indicate whether arc \((j, k)\) is connected in the network \( G(V^b, A^b) \). The TSE network \( G(V^b, A^b) \) of bus type \( i \) can be represented by the following equations:

\[
\begin{align*}
  u_{jk}^i & = 1 \quad \forall j \in O^b, k \in \{k | k \in T^b, e_i^k = 0\} \\
  u_{jk}^i & = 1 \quad \forall j \in \{j | j \in T^b, e_i^j > 0\}, k \in O^b \\
  u_{jk}^i & = 1 \quad \forall j \in T^b \cup F, k \in \{k | k \in T^b, \tau_i^k = \tau_i^j + t_{jk}^i, e_i^k = e_i^j + q_{jk}^i, e_i^k \leq \sigma Q_i\} \\
  u_{jk}^f & = 1 \quad \forall j \in T^b, \sigma_i Q_i \leq e_i^j \leq \sigma Q_i, f \in \{f | f \in F, \tau_i^f = \tau_i^j + t_{jk}^f, e_i^f + q_{jk}^i \leq \sigma Q_i, e_i^f = 0\}
\end{align*}
\]
Let the fixed time step and energy step to be \( \xi \) and \( \zeta \). We first calculate the exact arrival time \( t^k_i \) and cumulative energy consumption \( e^k_i \) of each node, and put it into the corresponding time interval and energy levels in the network, i.e. \([t^k_i/\xi]\) and \([e^k_i/\zeta]\).

Constraints (1) and (2) specify the deadheading arcs from or to the depots, where they flow into the termini nodes with a zero energy consumption and flow from the nodes with a cumulative energy consumption larger than zero. Let \( \sigma \) be the reduction factor of the energy capacity for planning purposes, namely safe driving range rate, to avoid energy depletion of the bus in operation, say 80 or 90%. Constraint (3) defines the connectivity of trips to bus termini, and the arrival time and cumulative energy consumption of the destination node are computed. Meanwhile, it guarantees that the maximum energy capacity cannot be violated with a safe driving range rate \( \sigma \) to avoid stranding. All buses with low energy levels should still have enough energy to travel to a refueling station. To ensure this, we introduce a start refueling rate \( \sigma' \) (say, 10%), so buses with energy levels at or just above \( \sigma'Q^r \) should initiate refueling. Therefore, the range of energy for paying a visit to a refueling station is narrowed to \([\sigma'Q^r, \sigma Q^r]\), as shown in constraint (4), who handles the connectivity of visits to a refueling station. Let \( t^r_i \) be the refueling time for buses of type \( i \), a predetermined parameter, which can also be modeled as a function of the energy capacity, either linear or non-linear, or a fixed value if we use the charging technology of battery swapping. Constraint (4) also defines the refueling process, and resets the cumulative energy consumption to be zero. Notice that there is only one energy level at each time interval at each refueling station, i.e. \( e^r_i = 0 \). To further reduce the network size, we can add \( \sum_{p \in \mathcal{V}^j} u_{pj}^i > 0 \) in (2) to (4) for node \( j \) to avoid redundant connections if \( j \) is an isolated node without inflows.
2.2.2 Passenger TS network

The TS network of passenger flow is defined by a set of graphs $G(V^d, A^d)$, where $d$ refers to a particular OD pair belonged to the set of OD pairs $R$. Let $V^d$ be the set of nodes with $V^d = O^d \cup T^d \cup U^d$ where $O^d$ denotes the set of time-expanded passenger demands, and $U^d$ the set of unserved demands at the end of the daily service. $T^d$ is the set of time-expanded bus termini, where $T_1^d$ is the time-expanded departure set and $T_2^d$ is the time-expanded destination set with respect to OD pair $d$. Let $A^d$ be the set of arcs with three subsets: service arc set $S^d$, waiting arc set $W^d$, and detouring arc set $D^d$, i.e. $A^d = S^d \cup W^d \cup D^d$. In the detouring arc set, there are two types of sub-arc sets; one is walking arc set $D_1^d$, and the other is demand arc set $D_2^d$ such that $D^d = D_1^d \cup D_2^d$. Note that in each graph $G(V^d, A^d)$, there are three types of demand arcs: origin demand arc from nodes in $O^d$ to nodes in $T_1^d$, served demand arc from the last node in $T_2^d$ to node in $O^d$, and lost demand arc from the last nodes in $T^d \setminus T_2^d$ to $U^d$. Each service arc denotes a trip at a specific time, whose arc cost is the passenger traveling time cost. The flow on service arc denotes the number of onboard passengers, subject to the capacities of the bus services. The flow on the waiting arc, however, indicates the accumulated passengers not getting served and have to wait for the subsequent bus due to
insufficient capacity of the departed bus. Walking arc describes the movement of passengers between locations within walking distance. The origin demand arc carries the number of passengers that arrive at the departure terminus whereas the flows on served or lost demand arcs describe the number of served or unserved passengers at the end of the daily service. If serving all demand is important to the bus company, one may set a large penalty for the lost demand arc, so that more services will be provided to carry all the demand, at the expense of a higher operating cost. Let \( r_{jk}^{d} \) be an indicator specifying whether arc \((j,k)\) is connected in the network \(G(V^d, A^d)\). The TS networks of passenger flow are constructed as follows:

\[
\begin{align*}
    r_{jk}^{d} &= 1 \quad \forall d \in R, j \in O^d, k \in T_1^d \quad (5) \\
    r_{jk}^{d} &= 1 \quad \forall d \in R, j \in T_2^d, k \in O^d, (j,k) \in D_2^d \quad (6) \\
    r_{jk}^{d} &= 1 \quad \forall d \in R, j \in T_2^d \setminus T_1^d, k \in U^d, (j,k) \in D_2^d \quad (7) \\
    r_{jk}^{d} &= 1 \quad \forall d \in R, j \in T_2^d, (j,k) \in A^d \setminus D_2^d \quad (8)
\end{align*}
\]

2.3 Mathematical formulation

The MD-MVT-VSP based on TSE network can be stated as follows:

Let \((P1)\):

\[
\begin{align*}
    \min_{w,x,y} & \sum_{i \in I} \sum_{j \in O^d} V_i^j Y_i^{jk} + \sum_{i \in I} \sum_{j,k \in V^d} (C_j + E_i) q_i^j Y_i^{jk} + \sum_{i \in I} \sum_{g \in F} W_{ig} \\
    & + \sum_{d \in R} \sum_{j \in O^d} V_i^j X_j^d + \sum_{d \in R} \sum_{j,k \in V^d} V_i^j X_j^d \sum_{d \in R} Y_i^{jk} - \sum_{d \in R} Y_i^{jk} = 0 \quad \forall i \in I, k \in V^b \\
    & \sum_{j \in O^d} Y_i^{jk} \leq K_i \quad \forall i \in I, k \in T^b \quad (11) \\
    & \sum_{i \in I} Y_i^{jk} \leq c_{jk} \quad \forall j,k \in V^b, j \neq k \\
    & X_j^d = B_j^d \quad \forall j \in O^d, k \in T^d, d \in R \quad (13) \\
    & \sum_{j \in A^d} X_j^d - \sum_{k \in A^d} X_j^d = 0 \quad \forall k \in T^d, d \in R \quad (14) \\
    & \sum_{d \in R} X_d^{jk} \leq \sum_{i \in I} Y_i^{jk} \zeta_i \quad \forall (j,k) \in S^d \\
    & \frac{1}{\omega} \sum_{i \in I} \sum_{j \in T^d} Y_i^{jk} \leq W_{ig} \quad \forall i \in I, g \in F_i \quad (16)
\end{align*}
\]

\( W \) is binary, \( X, Y \) is integer

\( (16) \)
Three types of decision variables are set in this problem. Let \( Y = \{ Y_{jk}^i \} \) be the set of integer variables indicating the bus flow from node \( j \) to node \( k \) in the bus flow network \( G(V^b, A^b) \). Let \( W = \{ W_g^i \} \) be the set of binary variables denoting whether the refueling station \( g \) of bus type \( i \) is in use. Let \( X = \{ X_{jk}^d \} \) be the set of integer variables representing the passenger flow from node \( j \) to node \( k \) for OD pair \( d \) in the passenger flow network \( G(V^d, A^d) \).

The objective of minimizing the total system cost within the planning horizon is defined in (9), including the operator cost, passenger cost, and external cost of emissions. Let \( V^1_i \) and \( V^2_i \) be the fixed costs of owning each bus and each refueling station for the planning horizon. Let \( C_i \) be the operating cost of unit energy consumption associated with the fuel and maintenance costs, and \( E_{jk}^i \) be the external cost of emissions correspondingly. For each passenger, let \( V^s_{jk} \) be the monetary time cost corresponding to different arcs, where \( (j,k) \in A^d \setminus D^d_2 \), and \( V^u_{jk} \) be the penalty cost on the lost demand arc. The first three terms compose the operator cost and the external cost for the planning horizon, which are: (1) total fixed cost associated with owning all the buses; (2) total trip operating cost and external cost of emissions; and (3) total fixed cost of owning refueling stations. The last two terms refer to the passenger costs, which are: (1) total passenger traveling, waiting, and walking time costs; and (2) lost demand penalty.

As for the constraints, (10) represents the conservations of bus flows at each node \( k \) in the bus network \( G(V^b, A^b) \). Notice that assumption 1 in Section 2.1 is guaranteed when \( k \in O^b \). Constraint (11) ensures that the buses of type \( i \) in use should not exceed the maximum allowable fleet size \( K_i \) whereas (12) provides the road capacity \( c_{jk}^i \) on link \( (j,k) \). Constraint (13) assigns \( B^d_j \), passenger demand of node \( j \) on OD pair \( d \), to each departure terminus at which its conservations of passenger flows are guaranteed by (14). Let \( \zeta_i \) be the capacity of bus type \( i \). Constraint (15) requires that the total passenger volume served on arc \( (j,k) \) is subject to the total capacity of all types of buses on service arc \( (j,k) \). Constraint (16) defines the use of a refueling station if it is used at least once in all bus networks, where \( \sigma \) denotes an extremely large positive number. All in all, the formulation constitutes an ILP, which can solve not only the single fleet scheduling problem, but also the mixed fleet scheduling problem.
3 Experiments and results

To demonstrate the feasibility and optimality of the solutions, we conduct a numerical study with four depots, four termini, and four charging stations. Note that $P_1$ and $P_2$ are MILPs, and we use IBM ILOG CPLEX Optimization Studio 12.4 without one thread for solutions. The planning horizon consists of six 30-min intervals. For simplicity, we assume that passengers arrive at the beginning of each time interval.

As shown in Table 1, there are 3 OD demands. The travel distances of the four OD pairs are shown in Table 2 and the average vehicle speed is 23.5 km/h (Transport Department, 2014). The corresponding adjustment parameters for vehicle mass and speed are not considered in this example. Four candidate recharging stations for electric vehicles and four candidate depots are set at each terminus.

Li et al. (2012) used the pollution equivalent conversion values in China to estimate the emissions cost, whereas for the Hong Kong case in this paper, we use the external cost following the UK standard. Table 3 shows the bus emissions factors corresponding to the different energy sources, and the external costs of vehicle-related emissions. Notice that we include the indirect emissions from electricity generation for EBs in this study. The attributes of buses and other parameters, such as operating cost, purchasing cost, are shown in Table 4. It is worth noting that the refueling time for the electric bus is assumed to be a linear function with respect to the energy consumption for the first approach, where the function can be relaxed to any form in the future. Let the full refueling time for EBs to be 30 min in this case, the refueling time function is thus: \[ f\left(\epsilon_i \right) = \frac{30}{\sigma Q_i} \epsilon_i = \frac{30}{0.8*230} \epsilon_i = 0.163 \epsilon_i \]. The fixed cost of owning a bus refers to its purchase cost, whereas that of the refueling station refers to its construction cost. Both of them need to be transformed into the equivalent value of the planning period. For illustration purposes in this small example, we assume that a 40% subsidy is provided to EBs to enhance their financial feasibility. Meanwhile, we assume the energy capacity of EBs to be 16 kWh, and safe driving range rate to be 100% in this case. For each time interval of each bus terminus, energy consumption is divided into 5 levels by energy step of 4 kWh. According to Li and Ouyang (2011), the amortized annual construction cost of a charging station in China was estimated to be US$7500. The passenger value of time for 1 time interval (30 min) is US$ 0.3. The penalty of losing demand is US$ 10 per passenger.

Table 1

<table>
<thead>
<tr>
<th>OD</th>
<th>time</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tr>
<td>1(A-B)</td>
<td>72</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2(B-A)</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3(C-D)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4(D-C)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>70</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
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Table 2
Travel distance (km) of the small example

<table>
<thead>
<tr>
<th>O/D</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>10</td>
<td>0.8</td>
<td>12</td>
</tr>
<tr>
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</tr>
<tr>
<td>4</td>
<td>12</td>
<td>0.5</td>
<td>10</td>
<td>0</td>
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</tbody>
</table>

Table 3
Bus emissions factors and the external cost of vehicle-related emissions

<table>
<thead>
<tr>
<th></th>
<th>CO₂</th>
<th>CO</th>
<th>THC</th>
<th>NOₓ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diesel(g/L)</td>
<td>2600.32</td>
<td>5.57</td>
<td>2.71</td>
<td>24.20</td>
</tr>
<tr>
<td>Electric(g/kWh)</td>
<td>867.60</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>External cost (US$/g)</td>
<td>0.00002</td>
<td>0.00052</td>
<td>0.00101</td>
<td>0.00361</td>
</tr>
</tbody>
</table>

*aFrom Pelkmans et al. (2001)*

*bFrom Doucette and McCulloch. (2011)*

*cFrom Cen et al. (2016)*

Table 4
Parameters set for different bus types

<table>
<thead>
<tr>
<th></th>
<th>Electric bus</th>
<th>Diesel bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifespan (year)</td>
<td>12&lt;sup&gt;a&lt;/sup&gt;</td>
<td>17&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Energy consumption rate</td>
<td>1.2 kWh/km&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.63 L/km&lt;sup&gt;d&lt;/sup&gt;</td>
</tr>
<tr>
<td>Refueling time(min)</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>Passenger capacity</td>
<td>72&lt;sup&gt;e&lt;/sup&gt;</td>
<td>71&lt;sup&gt;e&lt;/sup&gt;</td>
</tr>
<tr>
<td>External cost</td>
<td>0.020 US$/kWh</td>
<td>0.15 US$/L</td>
</tr>
<tr>
<td>Operating cost</td>
<td>0.16 US$/kWh&lt;sup&gt;f&lt;/sup&gt;</td>
<td>1.27 US$/L&lt;sup&gt;g&lt;/sup&gt;</td>
</tr>
<tr>
<td>Purchase cost (US$)</td>
<td>790000&lt;sup&gt;b&lt;/sup&gt;</td>
<td>321143&lt;sup&gt;h&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

*aFrom Chicago Transit Authority (2017)*

*bFrom Legislative Council of Hong Kong (2014)*

*cFrom Nylund et al. (2012)*

*dFrom Pelkmans et al. (2001)*

*eFrom The Encyclopedia of Bus Transport in Hong Kong (2016)*

*fFrom Noel and McCormack (2014)*

*gFrom Clear et al. (2007)*

*hFrom Dickens et al. (2012)*

The scheduling results show that one electric bus is needed to carry all the trips with one charging action in the middle. Let the bus termini be represented by A, B, C, D, respectively, depot by O, and refueling station by RS, and the number beside denotes the time interval. For O and RS, their subscripts designate the exact
depot or refueling station identity. For example, A1 denotes terminus A at time 1, and RS,3 denotes the refueling station #4 at time 3, and O1 depot #1. The path for this electric bus is O1-A1-B2-D3-RS,3-RS,4-D4-C5-O1, as shown in Fig. 2 (left). By solving the problem, not only is the charging station located (RS,4 at terminus D), but also is the charging time precisely decided (time 3 to 4). As an essential part in the system cost, the passenger movements are optimized as well. For each OD demand, the passenger flow has three possible directions. Passengers may take the direct service line, wait for the next service trip, or detour to the nearby bus terminus and take a service trip with a similar direction to the destination. Fig. 2 (right) shows all passenger flows for each OD demand. Due to the low cost-effective of launching the bus service from B to A for the demand of 2 passengers, these 2 passengers from B detour from B to D and then C to A using the service trip from D to C.

Fig. 2. Optimal bus flow and passenger flow in the small case

4 Concluding remarks

In this paper, we formulated the multiple depot vehicle scheduling problem with multiple vehicle types (MD-MVT-VSP) under range and refueling constraints. Bus service coordination and the external cost of emissions were taken into consideration to generate a cost-effective and environmentally friendly bus scheduling scheme. To precisely solve the range and refueling issues, we developed a novel approach to generate the feasible time-space-energy (TSE) networks for each vehicle type. Likewise, a time-space (TS) network of passenger flow was generated to represent
the passenger movements. Based on these two kinds of feasible flow networks, a mixed-integer linear program (MILP) was developed to find the global optimal solution, which gives (1) the bus fleet size and its composition; (2) optimal service deployment schedule depicting the trip schedule as well as the vehicle schedule; (3) locations of the refueling stations; (4) route of each bus including the refueling activities; (5) passenger movements; and (6) total system cost including the operator cost, passenger cost, and the external cost of emissions. We then successfully applied the MD-MVT-VSP model to a small bus network and verified the effectiveness of this approach.

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